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On the transcendental nature of the class number formula

A classical problem in number theory is how to determine if two number fields are isomorphic. One approach is to verify whether the given number fields have the same arithmetical invariants (e.g., discriminant, signature, regulator, among others). In this talk, I will focus on the regulator, and show how much arithmetical information it encodes. More specifically, I will give, conditionally on the Weak Schanuel Conjecture, necessary and sufficient conditions for two totally real number fields to have the same regulator.

We will see that these conditions naturally appear from the following result: Two totally real number fields K_1 and K_2 have the same Dedekind zeta functions $\zeta_{K_1}(s)$ and $\zeta_{K_2}(s)$ if and only if the real numbers

$$\lim_{s\to 1}(s-1)\zeta_{K_1}(s) \quad \text{and} \quad \lim_{s\to 1}(s-1)\zeta_{K_2}(s)$$

are linearly dependent over the field of algebraic numbers $\overline{\mathbb{Q}}$. I will conclude with a representation-theoretic reformulation of this result.