JULIAN CHRISTOPHER, Wilfrid Laurier University

The effect of time delay on the global stability of infectious disease models and other dynamical systems

When studying mathematical models of infectious disease, one characteristic of great interest is the long term persistence of the disease: given an initial infection rate, will it become endemic? or die out? Mathematically this is a question about the stability of an endemic equilibrium. Since the initial conditions of an outbreak are not under our control, local stability results are often insufficient: one would like to know whether the conditions present in a specific outbreak would lead the system to the endemic equilibrium, or to disease extinction. For this reason, global stability results play an important role in the analysis of epidemic models.

In this talk we examine the effect of delayed variables on the global stability of systems of ordinary differential equations. This problem arises naturally in mathematical models of infectious disease, where delays may represent vector-borne transmission, incubation periods, temporary immunity, or other biological processes that prevent instantaneous responses in the system. Our approach is based on Lyapunov's Direct Method. Specifically, we construct Lyapunov functionals for the delayed system by modifying the Lyapunov functions used for the corresponding ODE model. This approach allows us to extend stability results from the non-delayed system to the delayed system, and provides a framework for analyzing the effect of delays in a broad class of dynamic models.