Student Research Sessions
Session de recherche des étudiants
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RUCHITA AMIN, Western University
Qualitative Dynamics of bifurcation Analysis on Immunotherapy of a Tumor Model with Treatment.

Among the recent advancements in cancer treatment, immunotherapy has emerged as a promising approach for managing and potentially curing malignant tumors. This work presents mathematical models to investigate the interactions between tumor cells, CD4+T cells, and cytokines, focusing on their role in tumor regression. The effectiveness of treatments involving CD4+T cells, cytokines, or polytherapy (a combination of both) is analyzed within this framework. The study identifies equilibrium points, examines solution stability, and conducts bifurcation analysis. Furthermore, the application of normal form theory provides insights into the amplitude, phase, and stability of limit cycles that arise from bifurcations. The research also examines the occurrence of multiple limit cycles driven by generalized Hopf bifurcations, leading to intricate dynamic behaviors. These findings indicate that Hopf bifurcations are a primary driver of oscillatory patterns, introducing a bistable configuration that includes both a stable limit cycle and a stable equilibrium. The implications of these results are discussed in the context of biological systems.

JÉRÉMY CHAMPAGNE, University of Waterloo
Equidistribution and the probability of coprimality in some integer tuples

"What is the probability of two random integers being coprime?" This question, sometimes called "Chebyshev's Problem", happens to have a very straightforward answer. Indeed, one can show with elementary methods that the natural density of pairs $(m,n) \in \mathbb{N}^2$ with $\gcd(m,n)=1$ is exactly $\zeta(2)^{-1} = 6\pi^{-2} \approx 60.8\%$. Knowing this, one might seek certain $g : \mathbb{N} \to \mathbb{N}$ for which the density of n's with $\gcd\big(n, g(n)\big) = 1$ is also $\zeta(2)^{-1}$, which gives a certain sense of arithmetic randomness to the function g. Many functions with that property can be found in the literature, and we have a special interest for those of the form $g(n) = \lfloor f(n) \rfloor$ where f is a real valued function with some equidistributive properties modulo one; for example, Watson showed in 1953 that $g(n) = \lfloor \alpha n \rfloor$ has this property whenever $\alpha \in \mathbb{R}$ is irrational. In this talk, I will give some intuition for results of this type and present some of my own contributions.

CHRISTINE EAGLES, University of Waterloo
Internality of autonomous algebraic differential equations

Sometimes, solutions to a system of differential equations can be considered as a subset of the constants of some differentially closed field of characteristic zero together with some fixed solutions. When this happens, we say the set of generic solutions is internal to the constants of this field. This talk describes progress, from joint work with Léo Jimenez, on developing an algebraic criterion for when solutions sets are almost internal to the constants.

LIAM GAUVREAU, University of Toronto
Symmetries in Tensor Spaces: An Introduction to Schur-Weyl Duality

Given a finite-dimensional vector space V, the groups S_n and $\mathrm{GL}(V)$ both have natural actions on the tensor space $V^{\otimes n}$ which commute. Schur-Weyl Duality asserts the images of these two actions in $\mathrm{End}(V^{\otimes n})$ are centralizers of each other. We discuss how the double centralizer theorem paired with this duality reveals a deep connection between the representation theory of the symmetric group and the general linear group, generalizing the well known decomposition $V \otimes V = \mathrm{Sym}^2 V \oplus \mathrm{Alt}^2 V$.

DONGLIN HAN, University of Alberta
Forecasting Seasonal Influenza Using Google Trends and Health-Seeking Behavior

Timely and accurate influenza forecasts are crucial for public health planning. In this talk, I will present a hybrid modeling approach that uses Google Trends data and historical health-seeking behavior to forecast seasonal flu activity. Our two-step method first predicts behavioral patterns with machine learning, then incorporates these forecasts into a mechanistic SIR model to forecast influenza trends. Results show that combining real-time internet search data with behavioral trends improves forecast accuracy, especially around peak seasons. This approach highlights the value of integrating digital and historical signals for enhanced infectious disease forecasting

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YUCEN JIN, Western University

The effect of delay on a Host-parasite model.

Latency is present in almost all real-world problems. However, the delay is often assumed to be negligible in many models in order to simplify analysis. In this presentation, a three-dimensional model that describes a Host, its competitor, and the parasite is analyzed, with the delay added in the production of the parasites. We will discuss how such latency would affect the dynamics of the model, including whether it would induce more complex bifurcations, and whether it would affect the coexistence of the species.

NGUYEN DAC KHOI NGUYEN, Memorial University

Surface area measures of α-concave functions and their Minkowski problem

An α-concave function serves as a functional generalization of convex bodies, which are main objects in the field of Convex Geometry. Among the key tools for studying convex bodies is the surface area measure, a measure on the sphere that contains the information of surface area of the convex body. In this talk, we first discuss the extension the concept of surface area measures from convex bodies to those of α-concave functions via a variational formula for their total mass.

The classical Minkowski problem asks for necessary and sufficient conditions for a Borel measure on the unit sphere to be the surface area measure of a convex body, and was first studied by Minkowski in 1897. Having defined the surface area measures of α-concave functions, it is natural to consider the Minkowski problem in this generalized setting. Notably, the main tool employed in addressing this problem is optimal transport, which is not a traditional approach to such problems

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RAHUL PADMANABHAN, Concordia University

Deep Learning Approximation of Matrix Functions: From Feedforward Neural Networks to Transformers

Deep Neural Networks (DNNs) have been at the forefront of Artificial Intelligence (AI) over the last decade. Transformers, a type of DNN, have revolutionized Natural Language Processing (NLP) through models like ChatGPT, Llama and more recently, Deepseek. While transformers are used mostly in NLP tasks, their potential for advanced numerical computations remains largely unexplored. This presents opportunities in areas like surrogate modeling and raises fundamental questions about AI's mathematical capabilities.

We investigate the use of transformers for approximating matrix functions, which are mappings that extend scalar functions to matrices. These functions are ubiquitous in scientific applications, from continuous-time Markov chains (matrix exponential) to stability analysis of dynamical systems (matrix sign function). Our work makes two main contributions. First, we prove theoretical bounds on the depth and width requirements for ReLU DNNs to approximate the matrix exponential. Second, we use transformers with encoded matrix data to approximate general matrix functions and compare their performance to feedforward DNNs. Through extensive numerical experiments, we demonstrate that the choice of matrix encoding scheme significantly impacts transformer performance. Our results show strong accuracy in approximating the matrix sign function, suggesting transformers' potential for advanced mathematical computations.

ANSH SHAN, Brock University

Linear Forms in Logarithms and Their Applications to Diophantine Equations

The theory of linear forms in logarithms is a fundamental tool in number theory, providing effective bounds on the solutions of Diophantine equations. In this talk, we will discuss the general framework of linear forms in logarithms, with an emphasis on the explicit bounds they establish. We will then explore the Baker-Davenport reduction method, which enables a significant refinement of these bounds, making them more practical for explicit computations. Finally, we will illustrate how these techniques facilitate the resolution of classes of Diophantine equations, demonstrating their effectiveness in proving finiteness results and determining explicit solutions. This talk aims to provide a comprehensive yet accessible discussion of these methods, appealing to students and researchers interested in number theory.

IVAN SHEVCHENKO, University of Toronto & University of Waterloo
Stability of Planar Switched Linear Dynamical Systems

Switched dynamical systems arise when we can arbitrarily transition between different subsystems, typically modelled by autonomous ordinary differential equations (ODEs). A fundamental example involves switched systems where all subsystems are linear. Studying their stability under arbitrary switching is important for practical applications. However, even for switched linear systems, there is no set of necessary and sufficient conditions to guarantee stability under arbitrary switching for arbitrary dimension of the phase space. In this talk, we present new necessary and sufficient conditions for ensuring uniform asymptotic stability of the origin under arbitrary switching in two-dimensional switched linear systems.

ZHEN SHUANG, Memorial University of Newfoundland
Hyperbolic Riesz potentials-capacities

We explore various energy estimates and optimal strong-weak embeddings for the hyperbolic Riesz potential, which is equivalent to the fractional wave operator $\Box^{\alpha}$. Additionally, we examine the hyperbolic Riesz capacity associated with this potential, focusing on its fundamental properties, its relationship to Lebesgue and Hausdorff measures, and its dual capacity.

DENYS SVETELIK, Concordia University
Geometric properties of the monoid $M = \langle L, R \mid LR^2 = RL^2 \rangle$

We discuss geometric properties of the Cayley graph of the monoid $M = \langle L, R \mid LR^2 = RL^2 \rangle$. In this talk, we will describe the distance function on the graph, highlighting its recursive structure, while also characterizing the horofunction boundary, which exhibits a hybrid topology: a Cantor-like set with isolated points situated between each pair of breakpoints.

This monoid M is notable as the first known hyperbolic monoid whose horofunction boundary is not a Cantor set, offering a counterpoint to classical examples.

TIANXU WANG, University of Alberta
Derivations of Animal Movement Models with Explicit Memory

Highly evolved animals continuously update their knowledge of social factors, refining movement decisions based on both historical and real-time observations. Despite its significance, research on the underlying mechanisms remains limited. In this study, we explore how the use of collective memory shapes different mathematical models across various ecological dispersal scenarios. Specifically, we investigate three memory-based dispersal scenarios: gradient-based movement, where individuals respond to environmental gradients; environment matching, which promotes uniform distribution within a population; and location-based movement, where decisions rely solely on local suitability. These scenarios correspond to diffusion advection, Fickian diffusion, and Fokker-Planck diffusion models, respectively. We focus on the derivation of these memory-based movement models using three approaches: spatial and temporal discretization, patch models in continuous time, and discrete-velocity jump process. These derivations highlight how different ways of using memory lead to distinct mathematical models. Numerical simulations reveal that the three dispersal scenarios exhibit distinct behaviors under memory-induced repulsive and attractive conditions. The diffusion advection and Fokker-Planck models display wiggle patterns and aggregation phenomena, while simulations of the Fickian diffusion model consistently stabilize to uniform constant states.

XUYUAN WANG, University of Alberta
Detecting and Mitigating Non-Identifiability in Infectious Disease Modeling

Non-identifiability is a common issue in infectious disease modeling, complicating the parameter estimation process when certain model parameters cannot be uniquely determined. This problem arises when multiple parameter sets provide equally good fits to the available data yet yield inconsistent predictions, thereby undermining the predictive reliability of mathematical models. In this talk, I will present an efficient method for detecting non-identifiable parameters using Singular Value Decomposition and Variance Decomposition (SVD-VD) techniques, along with a statistical approach based on regularized regression to mitigate the effects of non-identifiability. It can be demonstrated that the regularized estimator ensures local uniqueness, leading to more stable and reliable predictions. Numerical examples will be provided to illustrate the effectiveness of the SVD-VD method in conjunction with the regularized estimator.