JÉRÉMY CHAMPAGNE, University of Waterloo

Equidistribution and the probability of coprimality in some integer tuples

"What is the probability of two random integers being coprime?" This question, sometimes called "Chebyshev's Problem", happens to have a very straightforward answer. Indeed, one can show with elementary methods that the natural density of pairs $(m,n) \in \mathbb{N}^2$ with $\gcd(m,n)=1$ is exactly $\zeta(2)^{-1} = 6\pi^{-2} \approx 60.8\%$. Knowing this, one might seek certain $g : \mathbb{N} \to \mathbb{N}$ for which the density of n's with $\gcd\big(n, g(n)\big) = 1$ is also $\zeta(2)^{-1}$, which gives a certain sense of arithmetic randomness to the function g. Many functions with that property can be found in the literature, and we have a special interest for those of the form $g(n) = \lfloor f(n) \rfloor$ where f is a real valued function with some equidistributive properties modulo one; for example, Watson showed in 1953 that $g(n) = \lfloor \alpha n \rfloor$ has this property whenever $\alpha \in \mathbb{R}$ is irrational. In this talk, I will give some intuition for results of this type and present some of my own contributions.