Recent progress in matrix, graph and operator theory / Progrès récents dans la théorie des matrices, graphes et opérateurs
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LUDOVICK BOUTHAT, Université Laval
A question of Erdős on extremal matrices

In a celebrated paper of Marcus and Ree, it was shown that if $A = [a_{ij}]$ is an $n \times n$ doubly stochastic matrix, then there is a permutation $\sigma \in S_n$ such that $\sum_{i,j=1}^{n} a_{ij}^2 \leq \sum_{i=1}^{n} a_{i,\sigma(i)}$. Erdős asked for which doubly stochastic matrices the inequality is saturated. Although Marcus and Ree provided some insight for the set of solutions, the question appears to have fallen into oblivion. Our goal in this talk is to present recent progress on the problem since 2023.

DOUG FARENICK, University of Regina
Matrix convexity and unitary dilations of Toeplitz-contractive d-tuples

A well-known theorem of P.R. Halmos concerning the existence of unitary dilations for contractive linear operators acting on Hilbert spaces is recast as a result for d-tuples of contractive Hilbert space operators satisfying a certain matrix-positivity condition. Such operator d-tuples satisfying this matrix-positivity condition are called, herein, Toeplitz-contractive, and a characterisation of the Toeplitz-contractivity condition is presented. The matrix-positivity condition leads to definitions of new distance-measures in several variable operator theory, generalising the notions of norm, numerical radius, and spectral radius to d-tuples of operators (commuting, for the spectral radius) in what appears to be a novel, asymmetric way. Toeplitz contractive operators form a matrix convex set, and a scaling constant c_d for inclusions of the minimal and maximal matrix convex sets determined by a stretching of the unit circle S^1 across d complex dimensions is shown to exist.

AVLEEN KAUR, The University of British Columbia
Linear algebraic aspects of ultraspherical spectral methods

Spectral methods solve elliptic partial differential equations (PDEs) numerically and offer spectral convergence, meaning the error decays exponentially when the solution is analytic. We present numerical schemes for solving linear time-dependent PDEs using the ultraspherical spectral method in both space and time, achieving spectral convergence in the full discretization. The resulting systems are sparse and well-conditioned due to the underlying recurrence relations and operator representations. We discuss the linear algebraic features of these systems, including condition number behavior and their implications for numerical stability and solver efficiency. We also compare their performance with classical spectral schemes and briefly explore the potential for parallelization in time through linear algebraic techniques.

MATTHEW KREITZER, University of Guelph
Methods to construct de Bruijn Rings using circulant matricies

A de Bruijn torus is a two dimensional extension of a de Bruijn sequence. For modern robotics, the full tori is not always needed. Exclusions of perodic windows leads to a more practical tori known as a de Bruijn Ring. We have developed novel methods to generate these de Bruijn rings using linear circulant matricies over finite fields, named Circulant Shifters. In this talk we review this generation method, as well as methods classify these shifters and families of these shifters that can generate de Bruijn rings.

SARAH PLOSKER, Brandon University
Quasiorthogonality of $*$-subalgebras

Two unital $*$-subalgebras $\mathcal{A}$ and $\mathcal{B}$ of M_n cannot be orthogonal since they both contain the identity. However, they can be thought of as $quasi$-orthogonal in a natural way: if $\mathcal{A} \cap \{I\}^{\perp}$ and $\mathcal{B} \cap \{I\}^{\perp}$ are orthogonal in the trace inner product. This motivates several equivalent formal definitions of quasiorthogonality. We investigate the quasiorthogonality of commutative $*$-algebras. We introduce the new notion of a 'quasiable' matrix, which allows us to derive a new matrix-theoretic technique to compute the quasiorthogonality measure between pairs of commutative algebras, and we show how this approach can be extended to the general non-commutative case.

This is joint work with Sooyeong Kim, David Kribs, Edison Lozano, and Rajesh Pereira.

PAUL SKOUFRANIS, York University
Non-Commutative Majorization

The notion of majorization of one self-adjoint $n \times n$ matrix by another is a very useful concept in matrix/operator theory. For example, a classical theorem of Schur and Horn states that a diagonal matrix D is majorized by a self-adjoint matrix B if and only if a unitary conjugate of B has the same diagonal as D. Some equivalent characterizations of A being majorized by B include there existing a doubly stochastic matrix that maps the vector or eigenvalues of B to the the vector or eigenvalues of A, tracial inequalities involving convex functions of A and B, and there exists a mixed unitary quantum channel that maps B to A.

Given the prevalence of qunatum information theory, the following is an interesting question in the context of matrix/operator theory: given m-tuples $A_1, \ldots, A_m$ and $B_1, \ldots, B_m$ of $n \times n$ matrices, can a mathematical condition be given for when there exists a unital quantum channel Φ such that $\Phi(B_k) = A_k$ for all k. In this talk, we answer this question using non-commutative Choquet Theory as developed by Davidson and Kennedy.

This talk is based on joint works with Kennedy.

PRATEEK KUMAR VISHWAKARMA, Université Laval
Chevalley operations on TNN Grassmannians

Suppose $\mathbf{X} = (x_{ij})$ is an $n \times n$ matrix of indeterminates, and consider the real span of products of complementary minors: $\det \mathbf{X}_{P,Q} \det \mathbf{X}_{P^c,Q^c}$, where $P, Q \subseteq \{1, 2, \ldots, n\}$ have the same cardinality. Rhoades and Skandera [*Ann. Comb.* 2005] showed that this space has dimension equal to the nth Catalan number and identified a basis given by (TNN) Temperley–Lieb immanants. An important application of their result is a classification of determinantal inequalities that arise as real combinations of such products of minors for TNN $\mathbf{X}$, governed combinatorially by noncrossing partitions.

We introduce *Chevalley operations* on index sets and show several applications. Using the seminal bidiagonal factorization theorem for TNN matrices, Chevalley operations yield an algorithmic framework that provides an alternative classification of the inequalities above, from a new perspective closely related to certain sequences of cluster mutations.

This framework yields a new proof of Lam's log-supermodularity of Plücker coordinates [*Current Develop. Math.* 2014], leading to several notable consequences: (a) each positroid cell in Postnikov's decomposition (2006) of the TNN Grassmannian forms a distributive lattice; (b) log-supermodularity implies numerical positivity in the main theorem of Lam, Postnikov, and Pylyavskyy [*Amer. J. Math.* 2007]; and (c) we obtain an independent proof of the coordinatewise monotonicity of ratios of Schur polynomials, originally established by Khare and Tao [*Amer. J. Math.* 2021], which plays a central role in establishing quantitative estimates for entrywise positivity preservers.

HARMONY ZHAN, Worcester Polytechnic Institute
New connections between quantum walks and graph spectra

Quantum walks are quantum analogous of random walks on graphs. For both continuous-time and discrete-time quantum walks, the behavior of the walk is determined by certain spectral properties of the underlying graph. In this talk, I will discuss some new combinatorial problems arising from quantum walks, motivated by recent work on quantum state transfer and mixing.

XIAOHONG ZHANG, Université de Montréal

Laplacian state transfer

Let X be a graph, and let H be a Hermitian matrix associated to X, which is usually taken to be the adjacency or Laplacian matrix. At time t, the transition matrix of the continuous quantum walk on X relative to H is $U(t) = exp(itH)$. If the initial state of the walk is given by a density matrix D (positive semidefinite matrix of trace 1), then the state $D(t)$ of the walk at time t is $D(t) = U(t)DU(-t)$.

For $a \in V(X)$, we use e_a to denote the vector in $C^{V(X)}$ taking value 1 on the a-th coordinate and 0 elsewhere. Vertex states transfer has been studied extensively. Chen and Godsil introduced and studied pair state transfer, where the density matrix is $D = 1/2(e_a - e_b)(e_a - e_b)^T$, a scaled Laplacian matrix of the graph on $V(X)$ with exactly one edge ab. Both types of states are pure (D is of rank 1). In this talk, we consider perfect state transfer between more general states, and give characterizations of when perfect state transfer occurs. Transfer between rational states (all entries of D are rational), in particular Laplacian states (D is a scaled Laplacian matrix) will be discussed.