Combinatoire algébrique et énumérative

(Org: **Samuele Giraudo** and/et **Jose Dario Bastidas Olaya** (Université du Québec à Montréal))

ANTOINE ABRAM, UQÀM - LACIM

Dimension of unicycle poset

It is known that for uniformly random poset, when there is a low pobability for edges to appear in the cover graph, almost surely their cover graph will be a collection of trees and unicycle graphs.

Motivated by the study of the dimension of random posets, it was conjectured by Bollobás and Brightwell in 1997 that a finite poset whose cover graph contains at most one cycle has order dimension at most 3.

In this talk, we will discuss about poset dimension and give the idea behind a proof of this conjecture.

Joint work with A. Segovia

SPENCER BACKMAN, University of Vermont

Higher-Categorical Associahedra

In 2017, Bottman introduced a family of posets called 2-associahedra as a tool for investigating functoriality properties of Fukaya categories, and he conjectured that they could be realized as face posets of convex polytopes. We introduce a family of posets called categorical n-associahedra, which naturally extend Bottman's 2-associahedra and the classical associahedra. Categorical n-associahedra give a combinatorial model for the poset of strata of a compactified real moduli space of a tree arrangement of affine coordinate subspaces. We construct a family of complete polyhedral fans, called velocity fans, whose coordinates encode the relative velocities of pairs of colliding coordinate subspaces, and whose face posets are the categorical n-associahedra. In particular, this gives the first fan realization of 2-associahedra. In the case of the classical associahedron, the velocity fan specializes to the normal fan of Loday's realization of the associahedron. Time permitting, we will discuss current investigations of projectivity of velocity fans. This is joint work with Nathaniel Bottman and Daria Poliakova.

NANTEL BERGERON, bergeron@yorku.ca

Vine model for double forest polynomials

Together with Lucas Gagnon, Philippe Nadeau, Hunter Spink, and Vasu Tewari, we introduced double forest polynomials in our study of equivariant quasisymmetric functions and their connections to geometry. In this talk, I will discuss the vine model, which provides a framework for computing double forest polynomials.

ELISABETH BULLOCK, Massachusetts Institute of Technology

Ehrhart series of alcoved polytopes

In this talk (based on joint work with Yuhan Jiang), I will describe a general method for computing the Ehrhart series of any alcoved polytope via a particular shelling order of its alcoves. As an application, we get a bijective proof of the formula for the Ehrhart h*-polynomial of the second hypersimplex $\Delta_{2,n}$ in terms of Nick Early's decorated ordered set partitions.

ANGELA CARNEVALE, University of Galway, Ireland

Coloured shuffle compatibility and Hadamard products

In this talk I will present recent work on coloured shuffle compatibility of permutation statistics and its applications to zeta functions in algebra. I will discuss how we extended recent work of Gessel and Zhuang, introducing shuffle algebras associated with coloured permutation statistics. Our shuffle algebras provide a natural framework for studying Hadamard products of certain rational generating functions. As an application, we will see how to explicitly compute such products in the context of so-called class- and orbit-counting zeta functions of direct products of suitable groups. This is joint work with V. D. Moustakas and T. Rossmann.

SERGI ELIZALDE, Dartmouth

A bijection for descent sets of permutations with only even and only odd cycles

It is known that, when n is even, the number of permutations of $\{1, 2, \ldots, n\}$ all of whose cycles have odd length equals the number of those all of whose cycles have even length. Adin, Hegedűs and Roichman recently found a surprising refinement of this equality, showing that it still holds when restricting to permutations with a given descent set J on one side, and permutations with ascent set J on the other. Their proof is algebraic and relies on higher Lie characters. It also yields a version for odd n.

In this talk we give a bijective proof of the refined identity. First, using known bijections of Gessel, Reutenauer and others, we restate it in terms of multisets of necklaces, which we interpret as words. Then, we construct a weight-preserving bijection between words all of whose Lyndon factors have odd length and are distinct, and words all of whose Lyndon factors have even length.

ALEJANDRO GALVAN, Dartmouth College

Enumeration, structure and generation of triangular partitions

A *triangular partition* is a partition whose Ferrers diagram can be separated from its complement (as a subset of $\mathbb{N}^2$) by a straight line. Having their origins in combinatorial number theory and computer vision, triangular partitions have been studied from a combinatorial perspective by Onn and Sturmfels, and by Corteel et al. under the name *plane corner cuts*, and more recently by Bergeron and Mazin in the context of algebraic combinatorics.

In this talk, we give a new characterization of triangular partitions and the cells that can be added or removed while preserving the triangular condition, and use it to describe the Möbius function of the restriction of Young's lattice to triangular partitions. We obtain a formula for the number of triangular partitions whose Young diagram fits inside a square, deriving, as a byproduct, a new proof of Lipatov's enumeration theorem for balanced words. Finally, we present an algorithm that generates all the triangular partitions of a given size, which is significantly more efficient than previous ones and allows us to compute the number of triangular partitions of size up to 10^5.

This is joint work with Sergi Elizalde.

YAN LANCIAULT, UQÀM

Une symétrie des mots de Christoffel

Les mots de Christoffel sont à l'intersection de plusieurs domaines des mathématiques. De la théorie des nombres jusqu'à la géométrie discrète, en passant par la combinatoire de Catalan, ils apparaissent naturellement à des moments saugrenus. Plusieurs définitions équivalentes ont été proposées au fil du temps, algorithmiques, géométriques ou utilisant la notion de palindrome, chacune servant ici et là dépendant de la nature du problème étudié. Nous en proposons une nouvelle qui s'attarde sur l'ensemble de leurs facteurs devant posséder une certaine symétrie. La condition présenté est directement nécessaire, mais une surprise apparaît pour la rendre suffisante.

Plaisirs et blagues seront fournis!

GAYEE PARK, Dartmouth College

Naruse hook formula for mobile posets

Linear extensions of posets are important objects in enumerative and algebraic combinatorics that are difficult to count in general. Families of posets like Young diagrams of straight shapes and d-complete posets have hook-length product formulas to count linear extensions, whereas families like Young diagrams of skew shapes have determinant or positive sum formulas like the Naruse hook-length formula from 2014. In 2020, Garver et. al. gave determinant formulas to count linear extensions of a family of posets called mobile posets that refine d-complete posets and border strip skew shapes. We give a Naruse type hook-length formula to count linear extensions of such posets by proving a major index q-analogue. We also give an inversion index q-analogue of the Naruse formula for mobile tree posets.

SASHA PEVZNER, Northeastern University
Fixed quotients of polynomial rings and Stanley–Reisner rings

Given a finite group action on a ring, the fixed quotient is a natural module over the ring of invariants. We will first discuss this module structure for the symmetric group with its usual action on the polynomial ring, focusing on homological results and patterns. We will then investigate the fixed quotient of a certain Stanley–Reisner ring, also carrying a symmetric group action, which is related to the polynomial ring via Groebner degeneration. This suggests a framework for relating the study of the Stanley–Reisner fixed quotient to that of the polynomial ring.

COLLEEN ROBICHAUX, UCLA
Signed puzzles for Schubert coefficients

We give a signed puzzle rule to compute Schubert coefficients. The rule is based on a careful analysis of a recurrence of Knutson. We use the rule to prove polynomiality of the sums of Schubert coefficients with bounded number of inversions. This is joint work with Igor Pak.

ANDREW SACK, University of Michigan
Lattices from pointed building sets

We introduce a novel combinatorial structure called a *pointed building set*, which can be viewed as a family of lattices equipped with compatibility relations. To each pointed building set $\mathbf{B}$, we associate a complete lattice $\mathbb{O}(\mathbf{B})$, referred to as the *ornamentation lattice* of $\mathbf{B}$.

This construction has already been proven useful in understanding the structure of three families of lattices: operahedron lattices, the affine tamari lattice, and hypergraphic posets of tree intervals.

We examine several natural classes of pointed building sets which recover classical lattices such as the Tamari lattice, the lattice of topologies ordered by refinement, and the lattice of naturally labeled partial orders. Furthermore, several theoretical directions are explored including inverse limits and group actions. Notably, this leads to a straightforward construction of inverse limits of Tamari lattices, yielding infinite analogs of the Tamari lattice.

KARTIK SINGH, University of Waterloo
The quasisymmetric Macdonald polynomials are quasi-Schur positive at $t = 0$

The quasisymmetric Macdonald polynomials $G_\gamma(X; q, t)$ are a quasisymmetric refinement of the $P_\lambda(X; q, t)$'s that specialize to the quasisymmetric Schur functions $QS_\gamma(X)$. We study the $t = 0$ specialization $G_\gamma(X; q.0)$, which can be described as a sum over weighted multiline queues. We show that $G_\gamma(X; q, 0)$ expands positively in the quasisymmetric Schur basis and give a charge formula for the quasisymmetric Kostka-Foulkes polynomials $K_{\gamma,\alpha}(q)$ in the expansion $G_\gamma(X; q, 0) = \sum_\alpha K_{\gamma,\alpha}(q) QS_\alpha(X)$.

HUNTER SPINK, University of Toronto
Equivariant Quasisymmetry, noncrossing partitions, and a quasisymmetric flag variety

We introduce a notion of quasisymmetry for polynomials in two sets of variables, and discuss the connections to noncrossing partitions and the construction of a new "Quasisymmetric flag variety".

Joint with Nantel Bergeron, Lucas Gagnon, Philippe Nadeau, and Vasu Tewari

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TIANYI YU, UQAM

Normal Crystals for symmetric Grothendieck Polynomials

Schur polynomials form a fundamental basis for symmetric polynomials. Motivated by geometry and representation theory, researchers have expanded various polynomials into the Schur basis positively, including (i) skew Schur polynomials, (ii) products of Schur polynomials, and (iii) Stanley symmetric polynomials. Normal crystals provide an elegant framework that effectively demonstrates these Schur expansions. Symmetric Grothendieck polynomials are non-homogeneous analogues of Schur polynomials, arising from the K-theory of flag varieties. Analogous expansions into symmetric Grothendieck polynomials have garnered significant attention over the past decades: Buch established the K-theoretic analogues of (i) and (ii), while Buch, Kresch, Shimozono, Tamvakis, and Yong resolved (iii). In this talk, we present an analogue of normal crystal theory, introducing a powerful new tool for establishing symmetric Grothendieck positivity. This framework not only recovers the three K-theoretic expansions mentioned above but also sheds light on related problems, including a conjecture of Ikeda and Naruse. This work is based on a joint work with Eric Marberg and Kam Hung Tong.