TONG ZHANG, Memorial University

Fractional type inequalities in fractional Sobolev spaces on homogeneous Carnot groups with applications

We investigate several fractional type inequalities in fractional Sobolev spaces on homogeneous Carnot groups. Initially, we establish the equivalence between two types of fractional Sobolev function spaces. Moreover, we demonstrate that the fractional Sobolev inequality on the Lorentz scale is valid in these spaces when $1 \leq p < \frac{Q}{\alpha}$. Regarding these fractional Sobolev functions, we prove a fractional Hardy type inequality as well. Furthermore, we establish a fractional Adams-Moser-Trudinger type inequality, which provides exponential integrability for the fractional Sobolev functions when $p = \frac{Q}{\alpha}$. Additionally, a fractional Poincaré type inequality is derived for $p > \frac{Q}{\alpha}$ and $p = \infty$. Meanwhile, the fractional Morrey inequality holds true for $p > \frac{Q}{\alpha}$ and $p = \infty$. Overall, these results provide a systematic generalization of classical inequalities to fractional Sobolev spaces on homogeneous Carnot groups, covering various ranges of p. As an initial application, we explore the existence of weak solutions to p-sub-Laplacian equation and inequalities with fractional order, establishing several significant results. This is a joint work with Jie Xiao.