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Matrix convexity and unitary dilations of Toeplitz-contractive d -tuples

A well-known theorem of P.R. Halmos concerning the existence of unitary dilations for contractive linear operators acting on Hilbert spaces is recast as a result for d -tuples of contractive Hilbert space operators satisfying a certain matrix-positivity condition. Such operator d -tuples satisfying this matrix-positivity condition are called, herein, Toeplitz-contractive, and a characterisation of the Toeplitz-contractivity condition is presented. The matrix-positivity condition leads to definitions of new distance-measures in several variable operator theory, generalising the notions of norm, numerical radius, and spectral radius to d -tuples of operators (commuting, for the spectral radius) in what appears to be a novel, asymmetric way. Toeplitz contractive operators form a matrix convex set, and a scaling constant c_d for inclusions of the minimal and maximal matrix convex sets determined by a stretching of the unit circle S^1 across d complex dimensions is shown to exist.