Positivity: inequalities and preserving transformations
(Org: **Shaun Fallat** (Regina) and/et **Prateek Kumar Vishwakarma** (Laval))

SUJIT SAKHARAM DAMASE, Indian Institute of Science Bangalore
Total-positivity transforms in one and several variables

The study of positivity preservers goes back to Schur (1911) and Pólya–Szegő (1925), who showed that every convergent power series, when applied entrywise to positive semidefinite matrices of all sizes, preserves positivity. The converse was shown by Schoenberg (1942) and Rudin (1959): there are no other such preservers. Many subsequent variants (via changing the domain) and extensions (multivariate case) exist; e.g. a recent work by Belton–Guillot–Khare–Putinar classified the multivariate transforms of matrices with prescribed negative inertias.

I will discuss the parallel problem of preserving totally nonnegative (TN) or totally positive (TP) kernels, on arbitrary totally ordered sets X, Y. We begin with the work of Belton–Guillot–Khare–Putinar (2023), where they classified the post-composition operators F that preserve TN/TP kernels of each specified order, and showed that such univariate preservers F are either constant or linear. We then move from preservers to transforms, and from one to several variables. Namely, we completely characterize the transforms $F[-]$ that send each tuple of TN/TP kernels of orders $k_1, \ldots, k_p$ on $X \times Y$ to a TN/TP kernel of order l, for arbitrary prescribed positive integers (or infinite) $k_1, \ldots, k_p, l$. This is joint work with Apoorva Khare.

NATHANIEL JOHNSTON, Mount Allison University
The Factor Width and Factor Width Rank of a Matrix

The "factor width" of a positive semidefinite matrix is the smallest positive integer k for which it can be written as a sum of positive semidefinite matrices that are each non-zero only in a single k-by-k principal submatrix. We explore numerous problems related to factor width, such as how we can bound it in terms of the matrix's eigenvalues, and we briefly describe some recently-discovered applications of this quantity in quantum information theory.

We also explore the "factor-width-k rank" of a matrix, which is the minimum number of rank-1 matrices that can be used in a matrix's factor-width-at-most-k decomposition. We show that the factor width rank of a banded or arrowhead matrix equals its usual rank, but for other matrices they can differ. We also establish several bounds on the factor width rank of a matrix, including a tight connection between factor-width-k rank and the k-clique covering number of a graph.

POORNENDU KUMAR, University of Manitoba
Schwarz Lemma and Multiplier Algebras on Complete Nevanlinna–Pick Spaces

Let $\mathbb{D}$ denote the open unit disk in the complex plane. A classical result in complex analysis, the Schwarz lemma, asserts that if $f : \mathbb{D} \to \mathbb{D}$ is holomorphic with $f(0) = 0$, then f can be written as

$$f(z) = z\, g(z),$$

where g is a holomorphic self-map of $\mathbb{D}$. The collection of all such self-maps forms the unit ball of the algebra of bounded holomorphic functions on $\mathbb{D}$, which is precisely the unit ball of the multiplier algebra of the Hardy space $H^2(\mathbb{D})$. This space has the Szegő kernel as its reproducing kernel:

$$k(z, w) = \frac{1}{1 - z\overline{w}}.$$

Thus, the Schwarz lemma is understood on the unit ball of the multiplier algebra of the Hardy space.

In this talk, we explore a generalization of this perspective by replacing the Szegő kernel with other reproducing kernels—either on $\mathbb{D}$ or on the unit ball in $\mathbb{C}^n$—that satisfy the *complete Nevanlinna–Pick property*. Specifically, we will discuss a version of the Schwarz lemma for the unit ball of the multiplier algebra associated with such spaces.

BENJAMIN LOVITZ,, Northeastern University

SARAH PLOSKER, Brandon University
k-locally positive semidefinite matrices, factor width, and spectral inequalities

A Hermitian matrix X is called k-locally positive semidefinite if every $k \times k$ principal submatrix of X is positive semidefinite. These matrices form exactly the dual cone of the set of k-incoherent quantum states, in other words, matrices having factor width at most k. We develop some bounds on the possible spectra of k-locally PSD matrices, and present a method for numerically constructing a k-locally PSD matrix with a given spectrum. We explore the connection to the concept of k-incoherent states from quantum information theory, as well as the connection to hyperbolicity cones. This is joint work with Nathaniel Johnston, Shirin Moein, and Rajesh Pereira.

KARTIK SINGH, University of Waterloo
Parametrizing the Grassmannian using pipe dreams

Grassmannian has various decompositions, each with its own interesting combinatorics. In this talk we will be focusing on the Deodhar decomposition of the Grassmannian. We motivate the decomposition, and give a parametrization of the components using Go-diagrams, which are a special class of pipe dreams. We will also discuss the applications of this parametrization in the problem of analyzing the closure of Deodhar components.

DANIEL SOSKIN, Institute for Advanced Study, Princeton

MAXIMILIAN TORNES, University of Manitoba
Weighted composition operators on Hilbert function spaces on the ball

A weighted composition operator on a reproducing kernel Hilbert space (RKHS) is given by a composition, followed by a multiplication. We characterize unitary and co-isometric weighted composition operators on large class of RKHS of holomorphic functions on the Euclidean unit ball of $\mathbb{C}^n$. This extends results of Martín, Mas and Vukotić from the disc to the ball. This is joint work with Michael Hartz.

PRATEEK KUMAR VISHWAKARMA, Université Laval
A century of entrywise positivity preservers: from classical foundations to finite field solutions

Entrywise transforms preserving positive semidefiniteness has a rich history spanning over a century. In 1911, Schur established that the entrywise product of positive semidefinite matrices remains positive semidefinite – a foundational result in matrix analysis. Building on this, Pólya and Szegő in 1925 observed that functions given by power series with nonnegative Maclaurin coefficients, now known as absolutely monotonic functions, preserve positivity when applied entrywise to matrices of all sizes. They also posed a fundamental question: can a non-absolutely monotonic function exhibit the same positivity preserving property?

A major breakthrough came in 1942 when Schoenberg proved that all continuous preservers must indeed be absolutely monotonic. In 1959, Rudin removed the continuity requirement and conjectured a complex analogue: the preservers on complex Hermitian matrices must be power series in z and $\overline{z}$ with nonnegative coefficients; ultimately resolved by Herz in 1963.

These major developments have since inspired a broader theory connecting matrix analysis with fields like metric geometry, high-dimensional statistics, combinatorics, and real/complex analysis. In recent years, significant progress is made to address a challenging refinement of Schoenberg's theorem to characterize positivity preservers on matrices of a fixed dimension, whose partial resolutions involve symmetric function theory and combinatorics.

In this talk, we will survey the evolution of this theory over the past century, culminating in recent results of the speaker with D. Guillot, H. Gupta, and C. H. Yip resolving the aforementioned refinement in the algebraic framework of finite fields, thereby opening new avenues in discrete settings.