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Extending an arithmetic universe by an object

The study of Grothendieck toposes and elementary toposes has long been a central focus of category theory. However, replacing elementary toposes by something else could offer a different approach to this theory, and using arithmetic universes – an idea of Steve Vickers – has recently found some success.

Traditionally, one studies a Grothendieck topos with respect to an elementary topos $\mathcal{S}$ called the *base*. A Grothendieck topos with respect to this base consists of another elementary topos with a bounded geometric morphism to $\mathcal{S}$. This approach has its drawbacks, though, since geometric morphisms are not the most natural notion of morphism between elementary toposes.

On the other hand, morphisms of arithmetic universes (AUs) are much closer to geometric morphisms. Vickers suggested that it might be possible to develop the theory of Grothendieck toposes using AUs. This idea has seen some success: for instance, Vickers recently developed an analogue for classifying toposes using AUs.

In this talk, I'll examine the special case of extending an AU $\mathcal{A}$ by a single indeterminate object X. I'll show that, just like for Grothendieck toposes, this yields the category $\mathrm{CoPsh}(\mathbf{Fin}_{\mathcal{A}})$ of internal copresheaves on finite sets. This is the first step in giving a more concrete description of classifying toposes over an arithmetic universe, analogous to the case of a base elementary topos.

This work is the result of PhD research under the supervision of Simon Henry and Philip J. Scott, and is based on a conjecture of Simon Henry.