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Categorical foundations of discrete dynamical systems

We present a categorical framework for studying discrete dynamical systems aimed at understanding modularity. The motivation for studying such systems comes partly from mathematical biology, where discrete systems such as Boolean networks, Cellular Automata, and Petri Nets are studied. These can be used to study, e.g., gene regulatory networks. One of the fundamental problems is identifying steady states in these networks. In a gene regulatory network, this could represent a stable set of gene expressions.

A discrete dynamical system is a set X with a function $f\colon X \to X$. Thus, the category of discrete dynamical systems is $\mathsf{Set}^{B\mathbb{N}}$. We can (functorially) associate to a discrete dynamical system its state space, which is a specific type of directed graph. To study state spaces, we introduce the notion of a cycle set. We can assign a cycle set to a directed graph. The composition taking a discrete dynamical system to its cycle set is a right adjoint, so in particular preserves limits. As a proof of concept of our methods, we give a conceptual proof of a generalization of a decomposition theorem of Kadelka et al. (2023).

This talk is based on joint work with D. Carranza, C. Kapulkin, R. Laubenbacher, and M. Wheeler.