MAX PETROWITSCH, Western University *Elementary* ∞-*Toposes from Type Theory*

Elementary toposes are categories that share many properties of the category of sets. Every elementary topos has an internal language, which is a version of typed intuitionistic higher-order logic obtained from the lattices of subobjects. The notion of elementary ∞ -topos generalises this concept to ∞ -categories. It is conjectured that the internal language of an elementary ∞ -topos is Homotopy Type Theory (HoTT), that is Martin-Löf Dependent Type theory with Π , Σ and intensional identity types satisfying the univalence axiom. Instead of the lattice of subobjects we have a universal ∞ -groupoid of all (small) objects with the structure of a space.

In the talk, I will introduce and motivate the notion of elementary ∞ -topos, and I will sketch the progress that has been made so far towards proving the conjecture. I will explain how HoTT presents such an elementary ∞ -topos via its syntactic category built from the syntax and rules of the type theory. First, I will use the fact that the syntactic category of HoTT has the structure of a tribe in the sense of Joyal. I will extend Joyal's theory of tribes by introducing the notion of a univalent fibration in a tribe. These fibrations exist in particular in the syntactic category of HoTT. In the second step, I will explain how each such tribe presents via its localisation an ∞ -category and if the tribe has enough univalent fibrations then this ∞ -category is an elementary ∞ -topos.