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Presheaf models of dependent type theory

Here is a fundamental question in categorical logic: given a categorical model E of a logical theory and small category D, is the category $[D, \mathsf{E}]$ of E-valued presheaves on D again a model of this logical theory? Such presheaf models are ubiquitous in categorical logic; for example, in topos theory, we use the category $[[1], \mathsf{Set}]$ to show that certain toposes do not satisfy the law of excluded middle.

When studying models of dependent type theory, we have a new parameter: thanks to the work of Voevodsky, we know that each such model is canonically equipped with a class of equivalences. It thus makes sense to consider a small category D also equipped with a class of equivalences and a subcategory of the presheaf category $[D, \mathsf{E}]$ consisting of those presheaves that take equivalences to equivalences. Such constructions are again of fundamental importance, as they were used for example in the proof of Voevodsky's conjecture on homotopy canonicity of homotopy type theory.

In this talk, I will report on joint work with Fiore and Li (arXiv:2410.11728) on identifying sufficient conditions on a category D with a class of equivalences so that $[D, \mathsf{E}]$ and its subcategory of equivalence-preserving presheaves again form a model of dependent type theory.