Complex analysis, Harmonic analysis and Operator theory

(Org: **Marcu-Antone Orsoni** (Université Laval) and/et **Pierre-Olivier Parisé** (Université du Québec à Trois-Rivières))

ABDEL RAHMAN AL-ABDALLAH, Brandon University

Levi Foliations on Homogeneous CR Manifolds

In this talk, we present recent results on the geometry of compact homogeneous Levi-flat CR manifolds, which carry a natural foliation. We show that, in the homogeneous setting, all leaves are biholomorphic and homogeneous under a complex Lie group action.

We analyze two main classes: parallelizable CR manifolds and projective orbits, and then unify these cases using the CR-normalizer fibration to obtain a classification in low codimension. Particular focus is given to the structure and classification of manifolds with dense leaves, supported by explicit examples.

SHAHRIAR ASLANI, University of Toronto

Normal singular orbits with minimal rank of a real-analytic D-Hamiltonian

It is known that the intersection of singular with normal geodesics of a sub-Riemannian manifold might not be empty. In this talk, we demonstrate that, after a generic conformal perturbation of the metric g, all singular curves of a real-analytic sub-Riemannian manifold (M,D,g) become strictly abnormal —assuming D a totally non-holonomic distribution of co-rank 1. Our proof relies on techniques of Hamiltonian dynamics and differential geometry.

We obtain the mentioned claim as a corollary of a more general statement about real analytic D-Hamiltonians (a quadratic D-Hamiltonian is nothing but a sub-Riemannian Hamiltonian). Given a real-analytic D-Hamiltonian H, we will show if (Q,P) is an orbit of H+u, where u is a generic real-analytic potential, then Q is not a D-singular curve with minimal rank.

ILIA BINDER, University of Toronto

Schrödinger operators with small quasiperiodic potentials: comb domains

In the talk, we discuss a characterization of the spectra of Schrödinger operators with small quasiperiodic analytic potentials in terms of their comb domains. We will also discuss action variables motivated by the integrable Korteweg-de Vries (KdV) system. The talk is based on joint work with D. Damanik, M. Goldstein, and M. Lukic.

ALEX BRUDNYI, University of Calgary

Ultrametric Spaces, Spaces of Balls, and Pluripotential Theory

In pluripotential theory, capacity provides a means of measuring the "size" of sets in a way that is compatible with their complex-analytic properties, making it an essential tool in many problems arising in the theory of functions of several complex variables. The notion of capacity originated in the theory of functions of one complex variable and was subsequently extended to the multivariable setting through the work of several distinguished mathematicians. In the higher-dimensional context, the principal quantities of interest include the Chebyshev constant, the Robin constant, and the transfinite diameter. The study of these quantities is intimately related to the behavior of certain extremal plurisubharmonic functions. In this talk, I introduce a novel approach to capacity theory based on the geometry of specific ultrametric spaces and the metric spaces formed by their unit balls. This framework provides new insights into classical problems and establishes connections with other areas of analysis and geometric function theory.

ALMAZ BUTAEV, University of the Fraser Valley

Discrete Dirichlet forms on a regular metric measure space

We will discuss a construction of discrete Dirichlet forms on Newtonian functions $N^{1,2}$ that are comparable to the upper gradient energy form. We will look into properties of the Γ-limit of these Dirichlet forms, and talk about the Dirichlet boundary value problem associated with it. This is joint work with N. Shanmugalingam and L. Luo.

KRZYSZTOF CIOSMAK, University of Toronto

Cartan–Thullen theorem and Levi problem in context of generalised convexity

In the talk I will introduce analogues of the classical notions of the complex analysis to the setting of generalised convexity and I will demonstrate that the Cartan-Thullen theorem and its appropriate formulation in the context of generalised convexity, which I will present, can be regarded as consequences of the classical theorems of functional analysis: the Banach-Steinhaus theorem and the Banach-Alaoglu theorem. Furthermore, I will provide a characterisation of the domains of holomorphy, and their generalisations, as the spaces that are complete, or as the spaces exhaustible by suitably defined polytopes. I will also provide an abstract analogue of the Levi problem and its elementary resolution. My results allow also for a novel characterisation of Stein spaces as the holomorphically complete spaces, as well as show that the Bremermann-Lelong lemma is equivalent to the positive answer to the Levi problem.

VISHWA DEWAGE, Clemson University

Understanding Toeplitz operators from a QHA perspective

As first observed by Fulsche, Werner's quantum harmonic analysis (QHA) provides an effective tool to study Toeplitz operators on the Fock space.

QHA on Bergman spaces is more challenging but still has interesting consequences. We discuss a few recent developments in the theory of Toeplitz operators, obtained via QHA. One of them being a simple intuitive proof of the Berger-Coburn theorem for boundedness of Toeplitz operators.

SETAREH ESKANDARI, Umeå University

The boundedness of Hankel forms and operators

In this research, we focus on characterizing the boundedness of the bilinear Hankel form and the Hankel operator in the context of the weighted Bergman spaces, where the weights satisfy an upper-doubling requirement. The Hankel form and Hankel operator are also connected to the problem of Hankel measures. Therefore, our next goal is to characterize p-Hankel measures for $p \leq 2$. The proofs rely on duality and factorization from the existing theory of weighted Bergman spaces and the recent results on two-weight fractional derivatives.

PAUL GAUTHIER, Université de Montréal

Zero-free approximation, universality and the Riemann hypothesis

The problem of approximation of functions on a compact subset $K \subset \mathbb{C}$ by polynomials zero-free on K is related to the Riemann Hypothesis. We show that there is no topological obstruction.

DAMIR KINZEBULATOV, Université Laval

Heat kernels of finite particles with critical attracting interactions

In this talk, I will present heat kernel bounds for some particle systems with strong attracting interactions, including celebrated Keller-Segel finite particles. In dimensions $d \geq 3$ we use Nash's method but with respect to suitable "desingularizing" weights. In dimension $d = 2$, which poses additional challenges, we adapt Nash's ideas to a non-local setting – despite the fact that the object under study is local – in order to effectively handle the singularity of the interaction kernel. Joint work with Sallah Eddine Boutiah.

POORNENDU KUMAR, University of Manitoba
On the Annihilator of a Pair of Commuting Contractions

Given a contraction T on a Hilbert space $\mathcal{H}$, the *annihilator* of T is defined as

$$\mathrm{Ann}(T) = \{f \in H^\infty(\mathbb{D}) : f(T) = 0\},$$

which is a weak-$*$ closed ideal in the Banach algebra $H^\infty(\mathbb{D})$ of bounded analytic functions on the open unit disc $\mathbb{D}$. When this ideal is non-trivial, Beurling's theorem asserts that it is generated by an inner function θ. To study such annihilators, one introduces the notion of the *support* of an inner function, defined as the set of points in $\overline{\mathbb{D}}$ where the function either vanishes or cannot be analytically continued through the point. This support encodes both spectral and geometric information about $\mathrm{Ann}(T)$: it coincides with the spectrum of T, and the zero set of θ corresponds precisely to the point spectrum of T.

In this talk, we will discuss the annihilator of a pair of commuting contractions on a Hilbert space, and examine how the notions of support and spectral theory relate in this multivariable setting.

This is ongoing joint work with Prof. Raphaël Clouâtre.

JAVAD MASHREGHI, Laval University
A Banach-Steinhause Type Theorem

We introduce the notion of an asymptotically equicontinuous sequence of linear operators, and use it to prove the following result. If X, Y are topological vector spaces, if $T_n, T : X \to Y$ are continuous linear maps, and if D is a dense subset of X, then the following statements are equivalent:

i) $T_n x \to Tx$ for all $x \in X$, and

ii) $T_n x \to Tx$ for all $x \in D$ and the sequence (T_n) is asymptotically equicontinuous.

MAËVA OSTERMANN, CNRS & Université de Lille
Hypercyclicity of Toeplitz operators

The study of Toeplitz operators from the point of view of linear dynamics began with a seminal work by Godefroy and Shapiro, in which they characterized when a Toeplitz operator on the Hardy space with anti-analytic symbol is hypercyclic (i.e., has a dense orbit). Shkarin later characterized the hypercyclicity of tridiagonal Toeplitz operators, and his result was extended first by Baranov and Lishanskii, and later by Abakumov, Baranov, Charpentier, and Lishanskii.

In this talk, I will discuss a new characterization of the hypercyclicity of Toeplitz operators that we obtained using a model theory developed by Yakubovich in the 1990s.

This is a joint work with E. Fricain and S. Grivaux.

THOMAS RANSFORD, Université Laval
Double-layer potentials, configuration constants and applications to numerical ranges

We consider estimates $\|p(T)\| \le K \sup_{z\in\Omega} |p(z)|$, where T is a Hilbert-space operator, p is a polynomial and Ω is a compact convex set containing the numerical range of T. We show that the well-known Crouzeix–Palencia bound $K \le 1 + \sqrt{2}$ can be improved to $K \le 1 + \sqrt{1 + a(\Omega)}$, where $a(\Omega)$ is what we call the analytic configuration constant of Ω. The latter is an analytic analogue of the classical configuration constant $c(\Omega)$ arising in the theory of double-layer potentials. A celebrated result of Neumann, dating back to 1877, states that $c(\Omega) < 1$ unless Ω is a triangle or a quadrilateral. Among other results, we prove that, in our case, we always have $a(\Omega) < 1$. Consequently, equality never holds in the Crouzeix–Palencia bound. (Joint work with Bartosz Malman, Javad Mashreghi and Ryan O'Loughlin.)

ERIC SAWYER, McMaster University
Trilinear characterizations of the Fourier extension conjecture on the paraboloid in three dimensions

This is joint work with Cristian Rios. We first prove that a local trilinear extension inequality on the paraboloid in three dimensions is equivalent to the Fourier restriction conjecture in three dimensions. Then we prove the equivalence of the above trilinear Fourier extension conjecture with the special case of testing a local trilinear inequality over certain smooth Alpert pseudoprojections, representing the weakest such inequality equivalent to the Fourier extension conjecture that the authors could find.

RASUL SHAFIKOV, University of Western Ontario
Meromorphic convexity on Stein manifolds

Rational convexity of compact subsets in complex Euclidean spaces is important in the approximation theory. I will discuss generalizations of rational convexity to Stein manifolds. I will then give a characterization of this type of convexity for a certain class of compacts in the spirit of Duval-Sibony, Guedj, and Nemirovski. This is joint work with B. Boudreaux and P. Gupta.

WILLIAM VERREAULT, University of Toronto
Fourier Decay of GMC Measures

The Fourier coefficients of multiplicative chaos measures appear naturally in the study of random matrices, QFTs, and even number theory. The harmonic analysis of the canonical GMC measure on the unit circle allowed Garban and Vargas to show that the associated Fourier coefficients tend to 0. The next step is to ask how fast this decay occurs, which corresponds to the Fourier dimension studied in fractal analysis. We compute the exact Fourier dimension of the circle-GMC measure, thereby proving a conjecture of Garban-Vargas based on a fourth moment computation. Our arguments are elementary, relying on a construction of an auxiliary, scale-invariant Gaussian field.

QUN WANG, University of Toronto Mississauga
The motion of two vortices in a simply connected domain

The motion of point vortices provides a finite dimensional approximation for the behavior of the incompressible, inviscid fluid. In this talk, after recalling how the dynamics of point vortices in a simply connected domain can be derived using conformal mapping techniques, we present some qualitative results concerning the dynamics of a system of two vortices in such domains. This work is part of an ongoing collaboration with J. Féjoz and M. Giralt.

MAHISHANKA WITHANACHCHI, University of Calgary
The Corona Problem on the Polydisk

In this talk, we study the Corona problem for the Banach algebra $H^\infty(\mathbb{D}^n)$ of bounded holomorphic functions on the polydisk $\mathbb{D}^n \subset \mathbb{C}^n$. In this setting, the Corona problem asks whether the polydisk $\mathbb{D}^n$ is dense in the Gelfand topology in the maximal ideal space of $H^\infty(\mathbb{D}^n)$.

We discuss certain cases of the Corona problem on the polydisk and present new necessary and sufficient conditions under which the problem can be solved. Our method is based on a new result concerning the solution of special $\bar{\partial}$ equations on a polydisk. This is joint work with Alex Brudnyi.

NINA ZORBOSKA, University of Manitoba
Berezin transform of some operators on weighted Hardy spaces

The Berezin transform is a function associated to an operator acting on a Reproducing Kernel Hilbert Space (RKHS) of functions. The properties of the operator affect the properties of the Berezin transform, and sometimes this also works in the other direction. We will illustrate some of this behaviour for a class of general Toeplitz operators acting on a range of RKHS called weighted Hardy spaces, which include the classical Bergman, Hardy and Dirichlet spaces. As it turns out, the Berezin transform approach works better for some of these spaces than for the others.