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Normal Crystals for symmetric Grothendieck Polynomials

Schur polynomials form a fundamental basis for symmetric polynomials. Motivated by geometry and representation theory, researchers have expanded various polynomials into the Schur basis positively, including (i) skew Schur polynomials, (ii) products of Schur polynomials, and (iii) Stanley symmetric polynomials. Normal crystals provide an elegant framework that effectively demonstrates these Schur expansions. Symmetric Grothendieck polynomials are non-homogeneous analogues of Schur polynomials, arising from the K-theory of flag varieties. Analogous expansions into symmetric Grothendieck polynomials have garnered significant attention over the past decades: Buch established the K-theoretic analogues of (i) and (ii), while Buch, Kresch, Shimozono, Tamvakis, and Yong resolved (iii). In this talk, we present an analogue of normal crystal theory, introducing a powerful new tool for establishing symmetric Grothendieck positivity. This framework not only recovers the three K-theoretic expansions mentioned above but also sheds light on related problems, including a conjecture of Ikeda and Naruse. This work is based on a joint work with Eric Marberg and Kam Hung Tong.