GABRIEL JARRY-BOLDUC, Mount Royal University

The cosine measure of a function

The cosine measure of a set of vectors is a valuable tool in derivative-free optimization (DFO) to judge the quality of the set. It provides information on how uniformly the set of vectors is covering the space $\mathbb{R}^n$. It is used in the convergence theory of certain DFO algorithms. A set of vectors is a positive spanning set of $\mathbb{R}^n$ if and only if its cosine measure is greater than zero. An important property of positive spanning sets is that when the gradient of a function at a point is well-defined and not equal to the zero vector, then there is at least one descent direction (ascent direction) of the function at the point contained in this set. This is not necessarily true if the gradient is equal to the zero vector or if the gradient does not exist. To characterize the previous two cases, the novel concept of cosine measure of a function is introduced in this paper. It provides an infimum on the value of the cosine measure of a set of vectors guaranteed to contain a descent direction of the function at the point of interest. Pseudo-codes are developed that makes it possible to compute the cosine measure of infinite sets of vectors and it is shown how to compute the cosine measure of a function for two popular classes of nonsmooth functions: indicator functions and max functions.