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*Characterisation of optimal solutions to second-order Beckmann problem through bimartingale couplings and leaf decompositions*

The aim of the talk is to present the three-marginal optimal transport problem - introduced in [K. Bolbotowski, G. Bouchitté, Kantorovich-Rubinstein duality theory for the Hessian, 2024, preprint], whose relaxation is the second-order Beckmann problem, and to give a complete characterisation of the optimal solutions for arbitrary pairs $\mu, \nu \in \mathcal{P}(\mathbb{R}^n)$ of absolutely continuous measures with common barycentre such that there exists an optimal plan with absolutely continuous third marginal.

In the talk, I will define the concept of bimartingale coupling for a pair of measures and establish several equivalent conditions that ensure such couplings exist. One of these conditions is that the pair is ordered according to the convex-concave order, thereby generalising the classical Strassen theorem. Another equivalent condition is that the dual problem associated with the second-order Beckmann problem attains its optimum at a $\mathcal{C}^{1,1}(\mathbb{R}^n)$ function with isometric derivative.

I will prove that the problem for the pair $\mu, \nu$ completely decomposes into a collection of simpler problems on the leaves of the $1$-Lipschitz derivative $Du$ of an optimal solution $u \in \mathcal{C}^{1,1}(\mathbb{R}^n)$ for the dual problem. On each such leaf the solution is expressed in terms of bimartingale couplings between the conditional measures of $\mu, \nu$, where the conditioning is defined relative to the foliation induced by $Du$.