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General symmetry multi-reduction method for partial differential equations with conservation laws

A powerful application of symmetries is finding symmetry-invariant solutions of nonlinear partial differential equations (PDEs). For a given symmetry, these solutions satisfy a reduced differential equation with one fewer independent variable. It is well known that a double reduction occurs whenever the starting nonlinear PDE possesses a conservation law that is invariant with respect to the symmetry.

Recent work has developed a broad generalization of the double-reduction method by considering the space of invariant conservation laws with respect to a given symmetry. In its simplest formulation, the generalization is able to reduce a nonlinear PDE in 2 variables to an ODE with m first integrals where m is the dimension of the space of invariant conservation laws. Nonlinear PDEs in 3 or more variables can be reduced to an ODE similarly by using an algebra of given symmetries. Importantly, the algebra does not need to be solvable.

The general method employs multipliers and is fully algorithmic. In particular, no a priori knowledge of conservation laws of the nonlinear PDE is necessary, and the multi-reduction is carried out in one step.

In this talk, a summary of the general multi-reduction method will be presented for obtaining invariant solutions of physically interesting PDEs. Examples will be shown for quadruple reduction from a single symmetry; complete integration from a solvable algebra in one step; reduction via a non-solvable algebra.