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mKdV Loop Travelling Waves and Interactions of Loop Solitons

The modified Korteweg-de Vries(mKdV) equation is an integrable non-linear evolution equation which has applications in modeling various physical phenomena. It also describes the curvature of curve which undergoes a certain non-stretching geometrical evolution in the Euclidean plane. This curve motion finds applications in various areas, such as describing the dynamics of inelastic rope, modeling the evolution of the boundary of vortex patch(swirling region) in thin, sheet-like layer of incompressible fluid, and understanding the behavior of electrons quantized in thin-layered materials by studying the boundaries of electron cloud densities under strong electromagnetic fields. This talk focuses on mKdV curve motions called loop solutions. One class arise from soliton, heavy-tail(rational), and periodic solutions of the mKdV equation. These loop solutions exhibit intriguing symmetrical shapes: the soliton and heavy-tail cases describe a single loop which is open, and asymptotically straight or circular, respectively; the periodic case describes both open and closed loops which can have multiple crossings. Additionally, a class of colliding loop solutions are obtained from the 2-soliton solution of the mKdV equation. The collisions show interesting interaction patterns. A summary of different types of patterns will be given by categorizing the various shapes that occur during the interaction, which depend on the speed ratio of the initial two loops. Analytical and numerical methods are employed to determine the loop solutions for both classes, as well as the conditions determining interaction type in the case of collision. These findings contribute to a deeper understanding of the mKdV equation and solitons