Student Research Session
Session de recherche des étudiants
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MATTHEW ALEXANDER, University of Regina
Categories Without Explicit Coherence

Category theory provides a unifying framework for studying a large variety of mathematical structures, by viewing them through the lens of objects and the morphisms between them. However there are naturally arising categories that contain even more data: higher order morphisms-between-morphisms. These are the focus of higher category theory. There are two standard ways of modeling higher categories. In the algebraic approach, compositions of morphisms are subject to "coherence conditions", which grow quickly in complexity, making working with these categories difficult in dimensions higher than 2 or 3. The geometric approach avoids explicit coherence conditions by viewing higher dimensional morphisms as higher dimensional topological spaces, and encodes the coherence conditions in the contractibility of these spaces. Unfortunately, reasoning about morphisms as high dimensional spaces can be quite delicate, and intuition quickly falls away.

In this talk we present a model for higher categories that avoids both explicit coherence conditions and dimensional growth of its morphisms, by relaxing the axioms of higher categories involving consistent choices of morphisms, to ones that only require the *existence* of morphisms. We will show how ordinary models of higher categories and their tools arise in this new setting and how certain constructions in category theory can be generalized to this model.

MANDANA BIDARVAND, University of Saskatchewan
Analyzing arrays of qubits via a multi-scale approach

A quantum metamaterial is an engineered structure whose modes of interaction with the environment depend on its quantum state. A prototypical example of such a material is a structure consisting of an array of qubits interacting with the electromagnetic field. Motivated by the challenges of analyzing such structures, we have developed a custom scale-based approach. It furnishes an alternative albeit formally equivalent model of quantum information. Its framework is naturally analytic, rather than linear-algebraic. It is especially well-suited for the study of the physics of finite as well as infinite arrays of qubits. Foundational to our approach are the Borel isomorphism and the multiresolution analysis in the Haar basis, both of which appear in classical mathematical literature in non-quantum contexts. We use them as devices that enable an identification between $L_2(0,1]$ and the Hilbert space of an infinite array of qubits. In the resulting framework, quantum operations and observables are represented through geometric integral operators. Prior studies demonstrated that in some cases the dynamics of qubit arrays is solvable in the sense that the spectra of crucial operators can be given explicitly. We extend those results and show a path to further systematic explorations. As an unexpected upshot, we observe that the fundamental concept of calculus is inherent in an infinite array of qubits; indeed, the antiderivative arises as a natural and indispensable operator in this context. In other words, if a mathematical structure encompasses a full theory of the infinite array of qubits, then it can support calculus.

SHANE J. CRERAR, University of Regina
Rank and Separability

The problem of determining whether a given quantum state is entangled is np-hard. By restricting the problem to extensions of a fixed faithful state, the determination of separability becomes a matter of finding the rank of associated operators. But that doesn't seem to make the calculations any easier. This talk will discuss the reasons why and the methods used, in particular how this extends the concept of Schmidt rank to mixed bipartite states.

ALEJANDRO SANTACRUZ HIDALGO, University of Western Ontario
Generalized monotone functions in measure spaces.

Monotone functions over the real numbers are very well-behaved compared to general measurable functions. Consequently, a wide variety of techniques and applications are in place for working with them. In this talk, we explore the notion of an ordered core, which allows us to define core decreasing functions and generalize monotone functions to general measure spaces without reliance on a strict ordering among elements.

Through various examples, we illustrate the versatility and adaptability of this generalized perspective on decreasing functions. Furthermore, we demonstrate its practical utility by exploring its connection to the study of abstract Hardy's inequalities. This approach provides a uniform treatment of many different types of Hardy operators. In particular, we use the theory of core decreasing functions to prove a new characterization for the boundedness of an abstract Hardy operator between L^1 to L^q with general measures.

JIAHUI HUANG, University of Waterloo
Arc-Floer conjecture for homogeneous isolated singularities

Given an isolated hypersurface singularity, one may associate to it algebraic invariants by studying the space of arcs and jets, or topological invariants via its Milnor fiber. The arc-Floer conjecture predicts an isomorphism between the cohomology of the contact loci of arcs and the Floer homology of iterates of the monodromy on the Milnor fiber. The case of plane curve singularities has been proven by de la Bodega and de Lorenzo Poza. In this talk we explain the history of this conjecture and present the first class of examples in higher dimensions, which are the homogeneous isolated singularities. This is joint work with de Lorenzo Poza.

ARNAUD NGOPNANG NGOMPE, University of Regina
Change of enrichment along a weak monoidal Quillen pair

This work is motivated by the observation that, considering the Dold–Kan correspondence $N : \mathrm{sMod}_R \cong \mathrm{Ch}_{\geq 0}(R) : \Gamma$, for a category $\mathcal{C}$ that is enriched, tensored, and cotensored over the category of simplicial (left) R-modules sMod_R (the category of non-negatively graded chain complexes of (left) R-modules $\mathrm{Ch}_{\geq 0}(R)$, respectively), the $\mathrm{Ch}_{\geq 0}(R)$-enriched category $N_*\mathcal{C}$ (the sMod_R-enriched category $\Gamma_*\mathcal{C}$, respectively) does not inherit a tensoring nor a cotensoring over $\mathrm{Ch}_{\geq 0}(R)$ (sMod_R, respectively). In this talk, we generalize this observation, and we give an insight of which properties are preserved and which are weakened after changing the enrichment of a $\mathcal{V}$-enriched model category $\mathcal{C}$ along a right weak monoidal Quillen adjoint $G : \mathcal{V} \to \mathcal{W}$.

MANIMUGDHA SAIKIA, University of Western Ontario
Multi-qutrit exact synthesis over Clifford+T

Unitary matrices in quantum computing play a similar role to Boolean functions in classical computing, meaning that quantum gates are represented by unitary matrices. For practical classical computers, we choose a set of special gates (known as a universal gate set) and make circuits using these gates to generate any other Boolean function. However, the quantum version of circuit synthesis is a bit more complicated. In this talk, we will introduce what a universal gate set means in quantum computing.

There are various universal gate sets for both single and multi-qubit (two-level quantum system) cases. Due to various advantages, researchers have a growing interest in finding universal gate sets for higher-level quantum systems. To this end, in our joint work with Kalra, Valluri, Winnick, and Yard, we present an algorithm to exactly synthesize a circuit corresponding to qutrit (3-level quantum system) unitaries using the multi-qutrit Clifford+T universal gate set.