
The Representation Theory and Geometry of Quantum Algebras

Théorie des représentations et géométrie des algèbres quantiques

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FRANCIS BISCHOFF, University of Regina

Castling Equivalence for Logarithmic Flat Connections

Castling is an operation on linear representations which arises from the phenomenon of dual Grassmanians. In this talk, I will explain its relevance to the problem of extending a logarithmic flat connection across the singular locus of a hypersurface.

EMILY CLIFF, Université de Sherbrooke

Quasi-universal sheaves and generic bricks

This is based on joint work with Colin Ingalls and Charles Paquette. For a quiver $Q = (Q_0, Q_1)$ and dimension vector $d = (d_i)_{i \in Q_0}$ we study a coarse moduli M space of quiver representations. Let d be the greatest common divisor of the numbers d_i . In the case that $d = 1$, it is known that M admits a universal family U of representations, and hence is a fine moduli space: that is, U is a sheaf of kQ -modules on M such that for every point $m \in M$ corresponding to a kQ -module V_m , the fibre U_m of U at m is isomorphic to the representation V_m . However, this fails when $d > 1$ (Reineke–Schröer, Hoskins–Schaffhauser); instead M admits a *quasi-universal* family \tilde{U} whose fibre \tilde{U}_m is isomorphic to a direct sum of copies of the representation V_m . In this talk, I will introduce the notion of twisted sheaves and sketch the construction of the sheaf \tilde{U} . I will explain how we can use the quasi-universal sheaf \tilde{U} to construct generic bricks for the path algebra kQ .

PETER CROOKS, Utah State University

Topological quantum field theories in the Moore–Tachikawa category

I will briefly review Moore and Tachikawa’s conjectural topological quantum field theory (TQFT), as well as the representation theory underlying its formulation. This will lead to an outline of recent, affirmative evidence for the conjecture. I will also detail a systematic association of TQFTs to Lie-theoretic data. A distinguished role will be played by the partial Grothendieck–Springer resolutions and their Poisson-geometric relatives. This represents joint work with Maxence Mayrand.

NOAH FRIESEN, University of Saskatchewan

Braid groups and Baxter polynomials

It is a classical result in representation theory that the braid group of a simple Lie algebra \mathfrak{g} acts on any integrable representation of \mathfrak{g} via products of exponentials of Chevalley generators of \mathfrak{g} . In this talk, we show that modifying this action induces an action on the commutative subalgebra $Y_{\hbar}^0(\mathfrak{g})$ of the Yangian. Dualizing this modified action gives us a new factorization of the Baxter polynomials of any irreducible representation of the Yangian, whose zeros encode information about the tensor products of such representations.

TERRY GANNON, U Alberta

The search for exotic vertex operator algebras

A frustrating aspect of our current understanding of vertex operator algebras (VOAs) is a lack of examples. More precisely, there are very few if any examples of VOAs which are independent of classical math like Lie algebras or lattices or finite groups. Is this because that is all the VOAs there are? Or are there whole worlds of new families of VOAs, and we are just too dumb (too classical) to find them? If we look at some shadows cast by VOAs (e.g. their tensor categories), we find several hints that

such exotic VOAs should exist, and in fact be numerous. In my talk I'll sketch this story, and apply it to the most interesting exotic candidate: the so-called Haagerup CFT, which has been pursued by both mathematicians and physicists for well over a decade.

NIKLAS GARNER, University of Washington, Seattle
Raviolo vertex algebras

I will describe work with B. Williams developing an algebraic structure modeling local observables in mixed holomorphic-topological quantum field theories in three dimensions. The resulting algebraic structure is directly analogous to a vertex algebra, but where holomorphic functions on a punctured complex curve are replaced by (derived) functions on a punctured 3-manifold that are constant along the leaves of chosen transverse holomorphic foliation. Time permitting, I will describe a construction appearing in work with S. Raghavendran and B. Williams on how Higgs and Coulomb branches of 3d $N = 4$ theories are encoded in this structure.

SACHIN GAUTAM, The Ohio State University
Lattice operators of quantum affine algebras

Let \mathfrak{g} be a finite-dimensional, simple Lie algebra over the field of complex numbers, and U be the quantum, untwisted affine algebra, associated to \mathfrak{g} . It is well known that the affine braid group of \mathfrak{g} acts on any integrable representation of U . In particular, one obtains an action of the coroot lattice of \mathfrak{g} on such a representation. In this talk, I will present an explicit formula for these lattice operators on finite-dimensional representations of U , in terms of the generators of its maximal commutative subalgebra in Drinfeld's loop presentation. This formula was obtained in a joint work in progress with V. Toledano Laredo.

NICOLAS GUAY, University of Alberta
Orthosymplectic Yangians

This talk will provide a summary of what is currently known about the representation theory of orthosymplectic Yangians, in particular the classification of their finite dimensional representations.

MENG GUO, University of Illinois Urbana-Champaign
On the spectrification of Khovanov arc algebras

Leveraging skew Howe duality, we show that Lawson-Lipshitz-Sarkar's spectrification of Khovanov's arc algebra gives rise to 2-representations of categorified quantum groups over \mathbb{F}_2 that we call spectral 2-representations. These spectral 2-representations take values in the homotopy category of spectral bimodules over spectral categories. We view this as a step toward a higher representation theoretic interpretation of spectral enhancements in link homology. Following this idea, we hope to use this idea to construct a spectrum whose homology realized the Blanchet-Khovanov algebra. This is an ongoing project with Anne Dranowski, Aaron Lauda, and Andrew Manion.

IVA HALACHEVA, Northeastern University
Bethe subalgebras of the Yangian $Y(\mathfrak{gl}(n))$, tame representations, and Gelfand-Tsetlin patterns

The Bethe subalgebras of the Yangian $Y(\mathfrak{gl}(n))$ form a family of maximal commutative subalgebras indexed by points of the Deligne-Mumford compactification of the moduli space $M(0, n+2)$. When considering a point C in the real locus of this parameter space, the corresponding Bethe subalgebra $B(C)$ acts with simple spectrum on a given tame representation of $Y(\mathfrak{gl}(n))$. This results in an unramified covering, whose fiber over C is the set of eigenlines for the action of $B(C)$. I will discuss the identification of each fiber with a collection of Gelfand-Tsetlin keystone patterns, which carry a $\mathfrak{gl}(n)$ -crystal structure, as well as the monodromy action realized by a type of cactus group. This is joint work with Anfisa Gurenkova and Leonid Rybnikov.

JONAS HARTWIG, Iowa State University

Generalized reduction algebras

Reduction algebras are known by many names, including step algebras, Mickelsson algebras, Zhelobenko algebras, and transvector algebras. They are constructed out of an algebra map $U(\mathfrak{g}) \rightarrow A$ from an enveloping algebra of a reductive Lie algebra to an associative algebra. There are also super and quantum analogs. Their defining property is that they act on the space of singular vectors V^+ in any A -modules V . They are therefore closely related to the branching rule $A \downarrow U(\mathfrak{g})$ and intertwining operators.

In this talk we present some recent work on a generalization of the notion of reduction algebras where the enveloping algebra can be replaced by some other algebra without a triangular decomposition, such as the quantum group $U_q(\mathfrak{so}_n)$.

VALERIO TOLEDANO LAREDO, Northeastern University

On the Finkelberg-Ginzburg monodromy conjecture

Ginzburg and Finkelberg defined a mirabolic \mathcal{D} -module on the product of $SL_n(\mathbb{C})$ and its vector representation and conjectured that its monodromy on the open stratum is a covariant representation of the affine Hecke algebra of type A_{n-1} . We compute this monodromy for all values of the parameters (θ, c) in rank 1, and outside an explicit codimension 2 set of values in ranks 2 and higher. This shows in particular that the Finkelberg-Ginzburg conjecture, which was known to hold for generic values of (θ, c) , fails at special values even in rank 1. Our main tools are Opdam's shift operators and Cherednik's intertwiners for the affine Weyl group, which allow for the resolution of resonances of the mirabolic connection. This is joint work with Robin Walters (Northeastern).

ALEXIS LEROUX-LAPIERRE, McGill University

Obstructions to quantization of MV cycles using limits of characters

The apparently elementary question of writing down perfect bases for the irreducible representations of semisimple Lie algebras is a problem which finds its source in surprisingly involved mathematical tools. Two such sources are a version of the geometric Satake equivalence (giving rise to the so-called Mirkovic-Vilonen bases) and a categorification of U_q^- using KLR algebras (giving rise to the so-called dual canonical bases). It has been shown that those two families of bases do not coincide, raising the question of understanding the change of basis matrix. We introduce an algebraic equivariant multiplicity for modules over truncated shifted Yangians through limits of characters, effectively providing a tool to study this change of basis. Moreover, we apply this new notion to study whether, for given MV cycle, there exists a module over a truncated shifted Yangian whose characteristic cycle is precisely this MV cycle. This is joint work with Joel Kamnitzer.

DINUSHI MUNASINGHE, University of Toronto

Schur Algebras in Type B

Summary: We compare two type B generalizations of the q -Schur algebra: the cyclotomic q -Schur algebra of Dipper, James, and Mathas, and the algebra of endomorphisms commuting with the natural generalization of the Hecke action to type B, introduced by Lai and Luo. By writing the latter algebra as an idempotent truncation of the former, we leverage its properties to establish cellularity and study the crystal graph structure of the simples of the endomorphism algebra, investigating parameter values at which these algebras are Morita equivalent.

SHIGENORI NAKATSUKA, University of Alberta

On the structure of W -algebras

The W -algebras are a basic class of vertex algebras that non-linearly generalize the affine Lie algebras and the Virasoro Lie algebra. They have appeared prominently in various areas of mathematics. In this talk, we discuss hidden relations among

W-algebras, which can be understood from various points of view: Whittaker models of p -adic group representations, boundary conditions on $N=4$ super Yang-Mills theories, and the affine Yangians.

WENJUN NIU, Perimeter Institute for Theoretical Physics
Yangians for Takiff Algebra and Spectral R matrix

Let \mathfrak{g} be a Lie algebra and $\mathfrak{d} := T^*\mathfrak{g} = \mathfrak{g} \ltimes \mathfrak{g}^*$, which we call the Takiff algebra of \mathfrak{g} . In this talk, I will explain how one can construct a natural quantization of $U(\mathfrak{d}[\hbar])$ as a Hopf algebra, which I will denote by $\mathcal{A}_\hbar(\mathfrak{d})$. This will be a Hopf algebra with an action of the translation operator T , and moreover possess a spectral R matrix $R(z)$, such that:

$$\tau_z \otimes 1(\Delta_\hbar^{op}) = R(z)(\tau_z \otimes 1\Delta_\hbar)R(z)^{-1},$$

where $\tau_z = e^{zT}$ and $R(z)$ satisfies spectral quantum Yang-Baxter equation. I will explain how this construction is inspired by the study of holomorphic topological twists of 4d $\mathcal{N} = 2$ theories, as well as the construction of Gautam-Toledano-Laredo-Wendlandt.

MANISH PATNAIK, University of Alberta
Metaplectic Groups and Quantum Groups

The study of Whittaker functions on metaplectic groups arose from a desire to understand certain phenomenon in analytic number theory (distributions of Gauss sums, moments of L -functions). Motivated by ideas in the geometric Langlands program, we explain how this Whittaker theory on a ℓ -fold cover of a p -adic groups can be connected to the representation theory of quantum groups at a ℓ -th root of unity.

Joint work with Valentin Buciumas.

THÉO PINET, Université Paris Cité and Université de Montréal
Inflations for representations of shifted quantum affine algebras

Fix a finite-dimensional simple Lie algebra \mathfrak{g} and let $\mathfrak{g}_J \subseteq \mathfrak{g}$ be a Lie subalgebra coming from a Dynkin diagram inclusion. Then, the corresponding restriction functor is not essentially surjective on finite-dimensional simple \mathfrak{g}_J -modules. In this talk, we will study Finkelberg-Tsybaliuk's shifted quantum affine algebras $U_q^\mu(\mathfrak{g})$ and the associated categories \mathcal{O}^μ (defined by Hernandez). In particular, we will introduce natural subalgebras $U_q^\nu(\mathfrak{g}_J) \subseteq U_q^\mu(\mathfrak{g})$ and obtain a functor \mathcal{R}_J from $\mathcal{O}^{sh} = \bigoplus_\mu \mathcal{O}^\mu$ to $\bigoplus_\nu (U_q^\nu(\mathfrak{g}_J)\text{-Mod})$ using the canonical restriction functors. We will then establish that \mathcal{R}_J is essentially surjective on finite-dimensional simple objects by constructing notable preimages (called *inflations*) and will use these preimages to deduce certain R -matrices and examples of *cluster structures over Grothendieck rings*.

SURYA RAGHAVENDRAN, Yale University
Towards a Dolbeault AGT correspondence

In seminal work, Grojnowski-Nakajima constructed an action of the Heisenberg algebra on equivariant cohomology of Hilbert schemes. I will describe two holomorphic factorization algebras in three complex dimensions that furnish higher dimensional uplifts of the Heisenberg and Virasoro vertex algebras respectively. Conjecturally, mode algebras of these factorization algebras act on coherent cohomology of moduli of twisted Higgs sheaves on surfaces, and in a particular example, the action admits a cohomological deformation to the one studied by Grojnowski-Nakajima. I will describe motivation and evidence for this conjecture, rooted in a new mathematical understanding of a nebulous superconformal field theory in six dimensions.

YVAN SAINT-AUBIN, Université de Montréal
Bound quiver algebras that are Morita-equivalent to the Temperley-Lieb algebras of type B

Bound quiver algebras are in a sense the simplest (non-semisimple) algebras: their simple modules are one-dimensional, and indecomposable projective and injective ones can be read from their quiver presentation. Finding a path algebra that captures the representation theory of another given algebra is however very difficult. The family of Temperley-Lieb algebras TLb_n of type B (also known as the blob algebras) has a rich representation theory and is related to several important ones in both mathematics and physics: the affine Temperley-Lieb, the cyclotomic affine Hecke and the KLR algebras. Using Elias-Soergel-Williamson diagrammatic calculus we obtain bounded quiver algebras that are Morita-equivalent to the blocks of the algebras TLb_n . This is work in progress with Alexis Leroux-Lapierre and Théo Pinet. The relations on the bound quiver were also checked independently by Philippe Petit using KLR diagrammatics.

HADI SALMASIAN, University of Ottawa

Mapping a quantum group into a quantum Weyl algebra and applications

The representation of a reductive Lie algebra on a polynomial space by differential operators plays a pivotal role in classical invariant theory. In this talk, we describe a quantum analogue of this idea. We present some results, a conjecture, and an application to the quantum First Fundamental Theorem. This talk is based on joint work with Gail Letzter and Siddhartha Sahi.

YORCK SOMMERHAUSER, Memorial University

Hopf Algebras, Cohomology, and Mapping Class Groups

It follows from the general principles of topological field theory that mapping class groups of surfaces act on certain spaces of homomorphisms between certain representations of factorizable ribbon Hopf algebras. These homomorphism spaces, the so-called spaces of conformal blocks, or briefly block spaces, can be viewed as cohomology groups of degree zero. In the talk, we explain how this construction can be extended to cohomology groups of higher degree. The talk is based on joint work with S. Lentner, S. N. Mierach, and C. Schweigert.

MAMORU UEDA, University of Alberta

Affine Yangians of type A and non-rectangular W -algebras of type A

We will talk about how to construct a homomorphism from the affine Yangian of type A to the universal enveloping algebra of a non-rectangular W -algebra of type A . This homomorphism is an affine analogue of the one given by De Sole-Kac-Valeri and is surjective in the rectangular case. It is constructed by using the coproduct for the affine Yangian of type A and the Miura map for a W -algebra. As a consequence, we can obtain the compatibility between the coproduct for the affine Yangian and the parabolic induction for a non-rectangular W -algebra are compatible through this homomorphism. We expect that this homomorphism will be helpful for the generalization of the AGT conjecture, which will give a geometric representation of a W -algebra of type A .

HARSHIT YADAV, University of Alberta

Rigidity of VOAs and their extensions

I will start by motivating the study of non-semisimple modular tensor categories (ns MTC) using logarithmic vertex operator algebras (log VOAs). Among the most well understood techniques of obtaining new log-VOAs is by VOA extensions. The categorical counterpart of VOA extensions is the local module construction. We prove that given a ns MTC and a suitable commutative algebra A , the category of local modules is a ns MTC. This generalizes the results of Kirillov-Ostrik. Applying our result to VOA extensions allows us to prove that the extension of strong rational (resp., finite) VOA is strong rational (resp., finite). This talk is based on joint works with Kenichi Shimizu, Thomas Creutzig and Robert Mcrae.