Operators, Matrices, and Analytic Function Spaces
Opérateurs, matrices et espaces de fonctions analytiques
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ILIA BINDER, University of Toronto
Harmonic measure: can it be computed?

This talk discusses using Computability in Analysis. More specifically, it is concerned with the computability of the harmonic measure of a given domain. It will partially answer two key questions - "What is the requisite knowledge about a domain to compute its harmonic measure?" and "Can one always use the same algorithm to compute the harmonic measure for all points of the domain?" The speaker will provide precise definitions and explore open computability questions in Complex Analysis. The talk is based on joint work with Adi Glucksam, Cristobal Rojas, and Michael Yampolsky.

LUDOVICK BOUTHAT, Université Laval
Matrix Norms Induced by Random Vectors

In a recent article, Chávez, Garcia and Hurley introduced a new family of norms $\|\cdot\|_{\mathbf{X},d}$ on the space of $n\times n$ complex matrices which are induced by random vectors $\mathbf{X}$ having finite d-moments. In this talk, the interesting properties of these norms are exhibited, and recent progress concerning the submultiplicativity of these norms is presented. In particular, we shall see that they are submultiplictive, as long as the entries of $\mathbf{X}$ have finite p-moments for $p=\max\{2+\varepsilon,d\}$.

DOUGLAS FARENICK, University of Regina
Operator systems of Laurent polynomials of bounded degree

A Fejér-Riesz operator system is a vector space, denoted $C(S^1)_{(n)}$ for a positive integer $n\geq 2$, of continuous complex-valued functions on the unit circle S^1 in the complex plane such that the Fourier coefficients $\hat{f}(k)$ of $f\in C(S^1)_{(n)}$ vanish for every integer k satisfying $|k|\geq n$. Thus, $C(S^1)_{(n)}$ is the space of Laurent polynomials of degree bounded above by $n-1$. The vector spaces $C(S^1)_{(n)}$ are function systems in the unital abelian C*-algebra $C(S^1)$ of all continuous $f: S^1\to\mathbb{C}$. In this lecture, I will consider $C(S^1)_{(n)}$ not as a function system, but as an operator system, thereby accessing the additional structure inherent to matrices over $C(S^1)_{(n)}$. The Toeplitz and Fejér-Riesz operator systems–the former being operator systems of Toeplitz matrices–are related in the operator system category through duality. Through duality, one obtains the C*-nuclearity of Toeplitz and Fejér-Riesz operator systems, as well as their unique operator system structures when tensoring with injective operator systems. I will also mention two applications: (i) a matrix criterion, similar to the one involving the Choi matrix, for a linear map of the Fejér-Riesz operator system to be completely positive; (ii) a completely positive extension theorem for positive linear maps of $n\times n$ Toeplitz matrices into arbitrary von Neumann algebras, thereby showing that a similar extension theorem of Haagerup (1983) for 2×2 Toeplitz matrices holds for Toeplitz matrices of higher dimension.

SHAFIQUL ISLAM, University of Prince Edward Island
Finite dimensional approximations of the Frobenius-Perron operator for piecewise convex maps with countable number of branches

Fixed points of the Frobenius-Perron operator of a dynamical system are stationary densities of invariant measures of the system. However, the Frobenius-Perron equation is a functional equation and it is difficult to solve. Using Ulam's method one can find finite dimensional approximations (Ulam's matrices) of the Frobenius-Perron operator. Ulam's matrices are stochastic matrices and their fixed points are approximations of the unique stationary density function of the system. In this talk, we consider a class of piecewise convex maps with countably infinite number of branches which possesses a unique stationary density f^* of an invariant measure. We develop an Ulam method for approximation of f^*. Convergence analysis is presented. We provide examples with errors between f^* and approximate stationary densities via Ulam's method.

MATTHEW KREITZER, University of Guelph
Matrix methods to construct De Bruijn Tori and Families

A de Bruijn torus is a two dimensional extension of a de Bruijn sequence. While methods exist to generate these tori, only a few such methods are known. One method involves using a generalization of de Bruijn sequences known as de Brujin families, however generation of these de Bruijn families is difficult. We have developed a novel method to generate de Bruijn families for an arbitrary alphabet and window size using certain matrices over finite fields known as Affine Shifters.

In this talk, we describe this novel generation method. We will also give an analysis on limitations with this generation method. Time permitting, we will describe their extension in generating de Bruijn families of higher dimension.

POORNENDU KUMAR, University of Manitoba
On Caratheodory's Approximation Theorem.

In 1926, Carathéodory, in his study of holomorphic functions from the open unit disc $\mathbb{D}$ of the complex plane to the closed unit disc $\mathbb{D}$, proved that any holomorphic self-map on $\mathbb{D}$ can be approximated by finite Blaschke products (uniformly on compact subsets). Afterward, Rudin generalized this result to the polydisc as well as the open unit ball.

In this talk, we will explore extended versions of this theorem, specifically Carathéodory's approximation theorem for matrix-valued functions on the disc, the bidisc, and multi-connected domains. Our discussion will primarily focus on two perspectives: one rooted in operator theory and the other viewed through the lens of operator algebra. We will delve into the limitations and benefits inherent in both approaches. Finally, we will see a few applications of this result.

JAVAD MASHREGHI, Laval University
An Application of Finite Blaschke Products in Numerical Range Studies

Let T be an operator on a Hilbert space H with numerical radius $w(T) \leq 1$. According to a theorem of Berger and Stampfli, if f is a function in the disk algebra such that $f(0) = 0$, then $w(f(T)) \leq \|f\|_\infty$. We give a new and elementary proof of this result using finite Blaschke products. A well-known result relating numerical radius and norm says $\|T\| \leq 2w(T)$. We obtain a local improvement of this estimate, namely,

$$\|Tx\|^2 \leq 2 + 2\sqrt{1 - |\langle Tx, x\rangle|^2}, \qquad (x \in H,\ \|x\| \leq 1),$$

Whenever $w(T) \leq 1$. Using this refinement, we give a simplified proof of Drury's teardrop theorem, which extends the Berger-Stampfli theorem to the case $f(0) \neq 0$.

Joint work with T. Ransford and H. Klaja

HRIDOYANANDA SAIKIA, University of Manitoba
A non-commutative boundary for the dilation order

Arveson's hyperrigidity conjecture focuses on the unique extension property (UEP) of representations of C^*-algebras with respect to a generating operator system. The states that are maximal in the dilation order fully encapsulate the cyclic representations of a C^*-algebra with the UEP. The set of all maximal states form a norm-closed set which remains stable under absolute continuity. In this talk, we will discuss an equivalent characterization of the dilation maximal states in terms of a *boundary projection*. Subsequently, we will state a reformulation of Arveson's hyperrigidity conjecture in terms of the non-commutative topological properties of this boundary projection. This is a joint work with Raphaël Clouâtre.

MAHISHANKA WITHANACHCHI, Laval University
Lebesgue Constants in Local Dirichlet Spaces

This study delves into the analysis of partial Taylor sums S_n, $n \geq 0$, as finite rank operators on any Banach space of analytic functions on the open unit disc. In the classical disc algebra setting, these operators are known as Lebesgue constants, with their precise norm remaining unresolved. However, our focus shifts to the local Dirichlet spaces $\mathcal{D}_\zeta$, where we accurately determine the norm of S_n. This exploration involves three distinct norms on $\mathcal{D}_\zeta$, each providing unique values for the norm of S_n as an operator on $\mathcal{D}_\zeta$. Notably, these findings stand in sharp contrast to the classical disc algebra. Moreover, we extend our investigation to Cesaro means σ_n on local Dirichlet spaces, aiming to precisely determine their norm for the three introduced metrics.

Lebesgue constants in local Dirichlet spaces are vital for guiding the selection of optimal finite-dimensional approximations in numerical solutions of partial differential equations with Dirichlet boundary conditions in mathematical physics.