DOUGLAS FARENICK, University of Regina

Operator systems of Laurent polynomials of bounded degree

A Fejér-Riesz operator system is a vector space, denoted $C(S^1)_{(n)}$ for a positive integer $n \geq 2$, of continuous complex-valued functions on the unit circle S^1 in the complex plane such that the Fourier coefficients $\hat{f}(k)$ of $f \in C(S^1)_{(n)}$ vanish for every integer k satisfying $|k| \geq n$. Thus, $C(S^1)_{(n)}$ is the space of Laurent polynomials of degree bounded above by $n-1$. The vector spaces $C(S^1)_{(n)}$ are function systems in the unital abelian C*-algebra $C(S^1)$ of all continuous $f : S^1 \to \mathbb{C}$. In this lecture, I will consider $C(S^1)_{(n)}$ not as a function system, but as an operator system, thereby accessing the additional structure inherent to matrices over $C(S^1)_{(n)}$. The Toeplitz and Fejér-Riesz operator systems–the former being operator systems of Toeplitz matrices–are related in the operator system category through duality. Through duality, one obtains the C*-nuclearity of Toeplitz and Fejér-Riesz operator systems, as well as their unique operator system structures when tensoring with injective operator systems. I will also mention two applications: (i) a matrix criterion, similar to the one involving the Choi matrix, for a linear map of the Fejér-Riesz operator system to be completely positive; (ii) a completely positive extension theorem for positive linear maps of $n \times n$ Toeplitz matrices into arbritary von Neumann algebras, thereby showing that a similar extension theorem of Haagerup (1983) for 2×2 Toeplitz matrices holds for Toeplitz matrices of higher dimension.