ZHENCHAO GE, University of Waterloo

Irregularities of Dirichlet L-functions and a parity bias in gaps of zeros

The integral of Hardy's Z-function from 0 to T measures the occurrence of its sign changes. Hardy proved that this integral is $o(T)$ from which he deduced that the Riemann zeta-function has infinitely many zeros on the critical line. A. Ivić conjectured this integral is $O(T^{1/4})$ and $\Omega_{\pm}(T^{1/4})$ as $T \to \infty$. These estimates were proved, independently, by M. A. Korolev and M. Jutila.

In this talk, we will show that the analogous conjecture is false for the Z-functions of certain "special" Dirichlet L-functions. In particular, we show that the integral of the Z-function of a Dirichlet L-functions from 0 to T is asymptotic to $c_\chi T^{3/4}$ and we classify precisely when the constant c_χ is nonzero. Experimentally, we find that the L-functions in this (thin) family have a significant and previously undetected bias in the distribution of gaps between their zeros. These phenomena appear to have an arithmetic explanation that corresponds to the non-vanishing of a certain Gauss- type sum.

This is joint work with Jonathan Bober and Micah Milinovich.