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Refinements of Artin's primitive root conjecture

Let $\mathrm{ord_p(a)}$ be the order of a in $(\mathbb{Z}/p\mathbb{Z})^*$. In 1927 Artin conjectured that the set of primes p for which an integer $a \neq -1, \square$ is a primitive root (i.e. $\mathrm{ord_p(a) = p - 1}$) has a positive asymptotic density among all primes. In 1967 Hooley proved this conjecture assuming the Generalized Riemann Hypothesis.

In this talk we will study the behaviour of $\mathrm{ord_p(a)}$ as p varies over primes, in particular we will show, under GRH, that the set of primes p for which $\mathrm{ord_p(a)}$ is "k prime factors away" from $p-1$ has a positive asymptotic density among all primes except for particular values of a and k. We will interpret being "k prime factors away" in three different ways, namely $k = \omega(\frac{p-1}{\mathrm{ord_p(a)}})$, $k = \Omega(\frac{p-1}{\mathrm{ord_p(a)}})$ and $k = \omega(p-1) - \omega(\mathrm{ord_p(a)})$, and present conditional results analogous to Hooley's in all three cases and for all integer k.

This is joint work with Leo Goldmakher and Greg Martin.