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Some old and new perspectives on the convergence of spectral methods

This talk will concern the convergence theory for Fourier and Chebyshev (ultraspherical) spectral methods for operator equations. The classical convergence theory typically succeeds by showing that the operator under consideration is relatively compact with respect to an operator that is sufficiently simple. In this vein, we discuss the results of G. M. Vainikko [Krasnosel'skii et al., 1972] and a modern reimplementation of the ideas for the convergence of the Fourier-Floquet-Hill method. The ideas are also applicable to the Riemann–Hilbert (Wiener–Hopf) problem on the circle. We then consider the convergence of Chebyshev collocation methods for boundary-value problems and use yet another result of G. M. Vainikko [Krasnosel'skii et al., 1972] to establish convergence of the rectangular collocation method [Driscoll and Hale, 2016], for a special class of boundary conditions. Lastly, building on these ideas, and the work of Olver and Townsend, we develop an ultraspherical collocation method for boundary-value problems that is provably convergent for all (reasonable) boundary conditions.