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Moduli Spaces of Weighted Stable Curves and their Fundamental Groups

The Deligne-Mumford compactification, $\overline{M_{0,n}}$, of the moduli space of n distinct ordered points on $\mathbb{P}^1$, has many well understood geometric and topological properties. For example, it is a smooth projective variety over its base field. Many interesting properties are known for the manifold $\overline{M_{0,n}}(\mathbb{R})$ of real points of this variety. In particular, its fundamental group, $\pi_1(\overline{M_{0,n}}(\mathbb{R}))$, is related, via a short exact sequence, to another group known as the cactus group. Henriques and Kamnitzer gave an elegant combinatorial presentation of this cactus group.

In 2003, Hassett constructed a weighted variant of $\overline{M_{0,n}}(\mathbb{R})$: For each of the n labels, we assign a weight between 0 and 1; points can coincide if the sum of their weights does not exceed one. We seek combinatorial presentations for the fundamental groups of Hassett spaces with certain restrictions on the weights. In particular, we express the Hassett space as a blow-down of $\overline{M_{0,n}}$ and modify the cactus group to produce an analogous short exact sequence. The relations of this modified cactus group involves extensions to the braid relations in S_n. To establish the sufficiency of such relations, we consider a certain cell decomposition of these Hassett spaces, which are indexed by ordered planar trees.