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Lie-Poisson Neural Networks (LPNets): Data-Based Computing of Hamiltonian Systems with Symmetries

Physics-Informed Neural Networks (PINNs) have received much attention recently due to their potential for high-performance computations for complex physical systems, including data-based computing, systems with unknown parameters, and others. However, applications of these methods to predict the long-term evolution of systems with little friction, such as many systems encountered in space exploration, oceanography/climate, and many other fields, need extra care as the errors tend to accumulate, and the results may quickly become unreliable. We provide a solution to the problem of data-based computation of Hamiltonian systems utilizing symmetry methods. Many Hamiltonian systems with symmetry can be written as a Lie-Poisson system, where the underlying symmetry defines the Poisson bracket. For data-based computing of such systems, we design the Lie-Poisson neural networks (LPNets). We consider the Poisson bracket structure primary and require it to be satisfied exactly, whereas the Hamiltonian, only known from physics, can be satisfied approximately. By design, the method preserves all special integrals of the bracket (Casimirs) to machine precision. LPNets yield an efficient and promising computational method for many particular cases, such as rigid body or satellite motion (the case of SO(3) group), Kirchhoff's equations for an underwater vehicle (SE(3) group), and others. We also discuss symmetry-reduced computations for cases of incomplete symmetry reduction, such as the dynamics of coupled rigid bodies.

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