BRENDAN ROONEY, Rochester Institute of Technology

Sparse Graphs with $q(G) = 2$

Given a graph G on n vertices, $\mathcal{S}(G)$ is the set of symmetric $n \times n$ matrices with the same off-diagonal zero pattern as the adjacency matrix of G. We say that a connected graph G has $q(G) = 2$ if there is a matrix $M \in \mathcal{S}(G)$ with exactly 2 distinct eigenvalues.

Recently, Barrett et al. (Barrett et al. *Sparsity of graphs that allow two distinct eigenvalues.* Linear Algebra Appl. 674(2023), 377–395) proved that graphs on n vertices with $q(G) = 2$ must have $|E(G)| \geq 2n - 4$. They also showed that the odd-order graphs with $q(G) = 2$ have $|E(G)| \geq 2n - 3$, and characterized the odd-order graphs that meet this bound. We complete the characterization of graphs with $|E(G)| = 2n - 3$ and $q(G) = 2$ by treating the even-order case. As part of our characterization, we resolve an open question of Barrett et al. by determining for each double-ended candle H, the sets of non-edges S for which $q(H + S) = 2$.