DARRYL HOLM, Imperial College London

Emergent singular solutions (ESS) in nonlinear wave PDEs

We discuss emergent singular solutions (ESS) in nonlinear wave PDEs.

(1) Start with asymptotic expansion for 1D shallow water waves.

(2) Identify the b-equation in n dimensions, H-Staley [2003]

Integrable in 1D, $b = 2$ Camassa-H [1993], $b = 3$ Degasperis-Processi [2002]

(3) Q: Why is $b = 2$ special? Is ESS is a property of integrability?

A: No. The ESS solution Ansatz is a momentum map, H-Marsden [2005]

(4) Are there other geodesic ESS with $b = 2$ in 1D? Yes! Fringer-H [2001]

(5) ESS for Stochastic CH? Yes! Crisan-H [2019] and Bendall-Cotter-H [2022]

(6) Are there ESS for $b = 2$ and $W^{1,r}$ norm? Cotter-H-Pryer [2023]

(7) Are there ESS embeddings for PDEs in 2D and 3D?. H-Staley [2004]