Association schemes and their applications
Les schémas d'association et leurs applications
(Org: **Allen Herman** (University of Regina), **Roghayeh Maleki** (University of Primorska) and/et **Andriaherimanana Sarobidy Razafimahatratra** (University of Primorska))

OWEN GOFF, University of Wisconsin Madison
A new perspective on the q-Onsager algebra and its presentations

When working with association schemes and distance-regular graphs, there arises an associated matrix algebra called the Bose-Mesner Algebra. The generators for this algebra satisfy a pair of tridiagonal commutator relations, and an affine transformation of these relations gives the q-Onsager algebra.

The q-Onsager algebra (O_q) contains several sets of recursively defined elements. Although O_q has only two generators, explicit polynomial forms for these elements are not yet known. Here we give an algebra called the quantum torus (T_q) and show that the images of many of these elements under a homomorphism to T_q have pleasing closed forms; this may allow for some mysteries about O_q to be solved.

HIMANSHU GUPTA, University of Regina
The least Euclidean distortion constant of a distance-regular graph

Embedding graphs into Euclidean spaces with least distortion is a topic well-studied in mathematics and computer science. Despite this research, there are just a few graphs for which the precise least distortion and a least distortion embedding is known. In 2008, Vallentin studied this problem for distance-regular graphs and obtained a lower bound for the least distortion of a distance-regular graph. In addition, he showed that this bound is tight for Hamming and Johnson graphs as well as strongly regular graphs and conjectured that his bound is always tight for distance-regular graphs. In this talk, we provide several counterexamples to this conjecture with diameter 4 and larger, but we also prove the conjecture for several families of distance-regular graphs. This is joint work with Sebastian M. Cioaba (University of Delaware), Ferdinand Ihringer (Ghent University), and Hirotake Kurihara (Yamaguchi University).

ALLEN HERMAN, University of Regina
The Terwilliger algebras of tournament and conference graph association schemes

In this talk we will consider the Terwilliger algebras of association schemes (X, S) of (odd) order n and class 2 that are self-dual. If $\{A_0 = I, A_1, A_2\}$ are the $n \times n$ adjacency matrices of one of these association schemes, then the graphs represented by A_1 and A_2 are cospectral, with odd valency $2u + 1$ in the non-symmetric doubly-regular tournament case and even valuency $2u$ in the symmetric conference graph case.

For a fixed vertex $x \in X$, and for $i \in \{0, 1, 2\}$, let $E_i(x)$ be the $n \times n$ diagonal matrix whose diagonal vector is equal to the x-th row of A_i. The Terwilliger algebra $T_x(S)$ of (X, S) at the vertex x is the algebra over $\mathbb{C}$ generated by the adjacency matrices of (X, S) together with its dual idempotents $E_0(x)$, $E_1(x)$, and $E_2(x)$. We will begin by showing how the dimension and irreducible modules of T_x are determined by the spectrum of $E_1(x)A_1E_1(x)$.

We will then consider the question of whether or not the full list $(T_x)_{x \in X}$ of Terwilliger algebras up to algebra isomorphism determines the association scheme (X, S) up to combinatorial isomorphism. For tournaments of order 27, the answer turns out to be NO when we consider the Terwilliger algebras over $\mathbb{C}$ but YES when we consider the Terwilliger algebras over $\mathbb{Q}$. To understand the latter, we use tools from rational representation theory, namely, Schur indices and fields of character values.

ROGHAYEH MALEKI, University of Primorska
On the Q-polynomial property of the full bipartite subgraph of a Hamming graph $H(D, n)$

The Q-polynomial property is an algebraic property of distance-regular graphs, that was introduced by Delsarte in his seminal work on association schemes and coding theory.

In 2023, Paul Terwilliger generalized the Q-polynomial property to graphs that are not necessarily distance regular. In [*J. Combin. Theory Ser. A*, 205:105872, 2024], it was shown that the Hasse diagrams of the so-called attenuated space posets, which can be viewed as the q-analogs of Hamming posets, are Q-polynomial. However, the Hasse diagrams of Hamming posets were not studied in the context of the Q-polynomial property. In this talk, I will show that these are also Q-polynomial.

VENKATA RAGHU TEJ PANTAGUI, University of Regina
Erdos-Ko-Rado type results in some Schurian Schemes

The classical Erdos-Ko-Rado (EKR) theorem and some of its variants can be viewed as characterizations of maximum independent sets of unions of graphs in some commutative Schurian (Orbital) schemes. For example, the classical EKR theorem characterizes independent sets of the Kneser graph, which is graph in the Johnson Scheme. Some other EKR-type results on vertices of Schurian schemes include the EKR theorem on subspaces and the EKR theorem on perfect matchings in a complete graph. Taking inspiration from algebraic proofs of the above-mentioned classical results, we attempt to identify Schurian schemes that are amenable to these algebraic methods and obtain some new EKR-type results in other domains.

ANDRIAHERIMANANA SAROBIDY RAZAFIMAHATRATRA, University of Primorska
On the smallest non-diagonalizable vertex-primitive digraphs

In 1980, Peter J. Cameron asked whether the adjacency matrices of arc-transitive digraphs are always diagonalizable. In 1985, Babai gave a negative answer to Cameron's question, and asked whether one can find vertex-primitive digraphs whose adjacency matrices are not diagonalizable. Recently, Li, Xia, Zhou, and Zhu gave an infinite family of such vertex-primitive digraphs. They further asked about the smallest such digraphs.

In this talk, I will show using the theory of commutative association schemes that the smallest vertex-primitive digraphs with non-diagonalizable adjacency matrices arise from the action of $\mathrm{PSL}_2(17)$ on the 2-subsets of the projective line $\mathrm{PG}_1(17)$, or equivalently on cosets of the dihedral group D_{16}.

ALYSSA SANKEY, University of New Brunswick
Strongly regular decompositions derived from regular two-graphs

A *strongly regular decomposition* is a strongly regular graph admitting a partition of the vertex set into two parts on which the induced graphs are strongly regular. These decompositions provide examples of strongly regular designs or SRDs. We investigate an infinite family of such graphs in which there are two possible decompositions, giving two SRDs, and show that existence of either one implies existence of the other. A graph in this family thus provides a single structure linking 5 (well, actually 7 as will be explained in the talk) strongly regular graphs.

PAUL TERWILLIGER, University of Wisconsin-Madison
The S_3-symmetric tridiagonal algebra

The tridiagonal algebra is defined by two generators and two relations called the tridiagonal relations. Special cases of the tridiagonal algebra include the q-Onsager algebra, the Onsager algebra, and the positive part of the quantum affine $\mathfrak{sl}_2$ algebra. In this talk, we introduce the S_3-symmetric tridiagonal algebra. This algebra has six generators. Any two generators commute or satisfy a pair of tridiagonal relations. The generators can be identified with the vertices of a hexagon, such that nonadjacent generators commute and adjacent generators satisfy a pair of tridiagonal relations. We show that any Q-polynomial distance-regular graph gives a finite-dimensional module for the S_3-symmetric tridiagonal algebra.

STEVEN WANG, Carleton University
On constructing bent functions from cyclotomic mappings

There have been many developments on construction of associate schemes using nonlinear functions such as p-ary bent functions. A Boolean function f in n variables with $f(0) = 0$ is bent if and only if the Cayley graph defined on $\mathbb{Z}_2^n$ by the support of a Boolean function is a strongly regular with parameters $(2^{2n}, 2^{2n-1} + \epsilon 2^{n-1}, 2^{2n-2} + \epsilon 2^{n-1}, 2^{2n-2} + \epsilon 2^{n-1})$, $\epsilon = \pm 1$. We study a new method of constructing Boolean bent functions from cyclotomic mappings. As a result, several new explicit infinite families of bent functions and their duals are derived.

MERI ZAIMI, Université de Montréal
Bivariate P- and Q-polynomial structures of association schemes based on attenuated spaces

Recently, bivariate and multivariate generalizations of the P- and Q-polynomial properties of association schemes have been proposed, and several examples of higher rank association schemes have been shown to fit within the generalized framework. In particular, bivariate P- and Q-polynomial structures have been obtained for association schemes based on attenuated spaces. While the bivariate P-polynomial structure can be analyzed using combinatorial arguments with the adjacency matrices, the same cannot be done with the bivariate Q-polynomial structure and the primitive idempotents, which complicates the proof. In this talk, I will explain how both the bivariate P- and Q-polynomial structures of association schemes based on attenuated spaces can be examined using recurrence relations and difference equations of the bivariate polynomials which form the eigenvalues and dual eigenvalues of the scheme. I will furthermore discuss the bispectral algebra associated to the bivariate polynomials as well as the Terwilliger algebra of the scheme.

HANMENG ZHAN, Worcester Polytechnic Institute
Generating quantum uniform mixing in association schemes

Some quantum algorithms require a "flat" input state, where all the entries have the same absolute value. This can be generated using certain quantum walks on graphs. In this talk, I will explain how properties of association schemes help us construct quantum walks that admit instantaneous uniform mixing and ϵ-uniform mixing.

XIAOHONG ZHANG, Université de Montréal
Multivariate P-polynomial association schemes and m-distance regular graphs

An association scheme is P-polynomial if and only if it consists of the distance matrices of a distance-regular graph. Recently, a generalization to multivariate P-polynomial association schemes has been proposed. In this talk, we will introduce m-distance-regular graphs and show their connection to multivariate P-polynomial association schemes.