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The Terwilliger algebras of tournament and conference graph association schemes

In this talk we will consider the Terwilliger algebras of association schemes (X, S) of (odd) order n and class 2 that are self-dual. If $\{A_0 = I, A_1, A_2\}$ are the $n \times n$ adjacency matrices of one of these association schemes, then the graphs represented by A_1 and A_2 are cospectral, with odd valency $2u + 1$ in the non-symmetric doubly-regular tournament case and even valuency $2u$ in the symmetric conference graph case.

For a fixed vertex $x \in X$, and for $i \in \{0, 1, 2\}$, let $E_i(x)$ be the $n \times n$ diagonal matrix whose diagonal vector is equal to the x-th row of A_i. The Terwilliger algebra $T_x(S)$ of (X, S) at the vertex x is the algebra over $\mathbb{C}$ generated by the adjacency matrices of (X, S) together with its dual idempotents $E_0(x)$, $E_1(x)$, and $E_2(x)$. We will begin by showing how the dimension and irreducible modules of T_x are determined by the spectrum of $E_1(x)A_1E_1(x)$.

We will then consider the question of whether or not the full list $(T_x)_{x \in X}$ of Terwilliger algebras up to algebra isomorphism determines the association scheme (X, S) up to combinatorial isomorphism. For tournaments of order 27, the answer turns out to be NO when we consider the Terwilliger algebras over $\mathbb{C}$ but YES when we consider the Terwilliger algebras over $\mathbb{Q}$. To understand the latter, we use tools from rational representation theory, namely, Schur indices and fields of character values.