GREGORY REID, University of Western Ontario

Differential-Algebraic and Numerical-Geometric Algorithms for Symmetries of Differential Equations

I give an overview of algorithms for symmetries and mappings. For example, if a differential equation R is polynomially nonlinear with rational coefficients, then the dimension and the structure of its Lie Algebra of symmetries can be determined algorithmically. Power series for the symmetries can be computed up to a given finite order, and sometimes exact symmetries are determined. Examples are given in Maple using the RIF differential-elimination algorithm and the Lie Algebra of Vector Fields (LAVF) packages.

Related questions concern those about the existence of mappings, e.g., of a given PDE to a more tractable one (e.g. nonlinear to linear), following work initiated by Bluman and Kumei. Other interesting algorithmic problems concern the computation of symmetry information for classes of differential equations.

Central to the above problems is the reduction of systems of differential equations to involutive forms that include their integrability conditions by sequences of prolongations and projections in Jet space. I will discuss developments in this area, including work by the speaker and collaborators.

PDE in applications often contains parameters and functions that are only known approximately. Thus, approximate versions of the above problems are of interest. For example, given a system R, when is R close to some R* with desirable symmetry properties (e.g. R* has a large symmetry group). Several approaches to this problem will also be discussed: exact and numerical. Numerical methods, including those from the new area of Numerical Jet Geometry, will also be discussed, including methods for DAE.