
RENATE SCHEIDLER, University of Calgary

The Ankeny-Artin-Chowla Conjecture in Actual and Fake Real Quadratic Orders

Quadratic orders exhibit vastly different structural and invariant properties, depending on whether the ambient quadratic field is real or imaginary. In an unpublished note from 2014, Henri Cohen made the surprising observation that a certain subring of an imaginary quadratic order where denominators are restricted to powers of one fixed prime behaves very much like a real quadratic order. Cohen coined the term "fake real quadratic order" for these special structures.

The somewhat controversial Ankeny-Artin-Chowla (AAC) conjecture asserts a certain divisibility condition about fundamental units in real quadratic orders of prime discriminant. Although no counterexamples have been found despite extensive computations, number theorists are divided over the truth of this conjecture. A closely related conjecture, due to Mordell, was recently established to be false by Reinhart who found a counterexample.

In the hopes that an investigation of a "fake" AAC analogue might shed light on the original AAC conjecture, we investigated AAC in fake real quadratic orders. In this talk, which is aimed at a general math audience, I report our findings, consisting of extensive numerical computations, heuristics and asymptotic results. This is joint work with Hongyan Wang (a former Master's student at University of Calgary), Florian Hess (University of Oldenburg, Germany) and Mike Jacobson (University of Calgary).