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Analyzing arrays of qubits via a multi-scale approach

A quantum metamaterial is an engineered structure whose modes of interaction with the environment depend on its quantum state. A prototypical example of such a material is a structure consisting of an array of qubits interacting with the electromagnetic field. Motivated by the challenges of analyzing such structures, we have developed a custom scale-based approach. It furnishes an alternative albeit formally equivalent model of quantum information. Its framework is naturally analytic, rather than linear-algebraic. It is especially well-suited for the study of the physics of finite as well as infinite arrays of qubits. Foundational to our approach are the Borel isomorphism and the multiresolution analysis in the Haar basis, both of which appear in classical mathematical literature in non-quantum contexts. We use them as devices that enable an identification between $L_2(0, 1]$ and the Hilbert space of an infinite array of qubits. In the resulting framework, quantum operations and observables are represented through geometric integral operators. Prior studies demonstrated that in some cases the dynamics of qubit arrays is solvable in the sense that the spectra of crucial operators can be given explicitly. We extend those results and show a path to further systematic explorations. As an unexpected upshot, we observe that the fundamental concept of calculus is inherent in an infinite array of qubits; indeed, the antiderivative arises as a natural and indispensable operator in this context. In other words, if a mathematical structure encompasses a full theory of the infinite array of qubits, then it can support calculus.