Set theory and its applications
Théorie des ensembles et ses applications
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DAVOUD ABDI, University of Calgary
Counterexample to Conjectures of Bonato-Tardif, Thomassé and Tyomkyn, Future Directions

Two structures R and S are called *equimorphic* when each embeds in the other; we may also say that one is a *sibling* of the other. Equimorphic finite structures are necessarily isomorphic, but this is no longer the case for infinite structures. For instance, the rational numbers, considered as a linear order, has continuum many siblings, up to isomorphism. Thomassé (2000) conjectured that a countable relation has either one, countably or continuum many siblings, up to isomorphism. There is a special case of interest stating that a relational structure of any cardinality has one or infinitely many siblings. This is connected to a conjecture of Bonato-Tardif stating that a tree has one or infinitely many siblings.

In this talk we introduce the conjectures mentioned and those structures for which the conjectures have been verified by giving historical progress. Then, we introduce a counterexample to the conjectures and state open problems in the sibling program.

KEEGAN DASILVA BARBOSA, Fields Institute
Box Ramsey and Canonical Partitions

The KPT correspondence gives a full characterization of the dynamics of automorphism groups of Fraïssé structures through finite combinatorics. There is still much open however on whether or not there is a full correspondence between big Ramsey degrees and topological dynamics. While a partial answer has been found by Zucker by considering structures that admit a big Ramsey structure, the question still remains open. Motivated by this problem, we aim to answer a related question. Namely, what are the necessary and sufficient conditions needed for a structure to admit a finite list of canonical relations? We do so by developing a natural productive analogue to big Ramsey we call the Box Ramsey degree, solving a question of Masulovic. Our techniques will be reminiscent of Rado's proof of the Erdös-Rado theorem, or more recently, works on canonical equivalence relations done by Laflamme, Sauer, and Vuksanovic.

RUIYUAN CHEN, University of Michigan
Quasi-treeable equivalence relations

A countable Borel equivalence relation is said to be treeable if there is a Borel assignment of a tree on each equivalence class. We prove various results showing that every Borel assignment of "large-scale approximate trees" can be turned into genuine trees, thereby yielding new sufficient criteria for treeability. Joint with Antoine Poulin, Ran Tao, and Anush Tserunyan.

CHRISTOPHER EAGLE, University of Victoria
Counting models of theories in non-first-order logics

In 1970 Morley proved that a countable first-order theory has either at most $\aleph_1$ many or exactly $2^{\aleph_0}$ many isomorphism classes of countable models, regardless of the value of $2^{\aleph_0}$. Ideas implicit in Morley's proof give the stronger fact that if a countable first-order theory has strictly more than $\aleph_1$ isomorphism classes of countable models then it has a perfect set of pairwise non-isomorphic countable models. We consider the possible number of isomorphism classes of countable models, and whether there are perfect sets of non-isomorphic models, for theories of several stronger logics (including second-order logic, logics with game quantifiers, and logics with partially ordered quantifiers). For second-order theories we show that the statement analogous to Morley's result is independent of ZFC. This talk is based on joint work with Clovis Hamel, Sandra Müller, and Frank Tall.

CHRISTOPHER KARPINSKI, McGill University
Hyperfiniteness of boundary actions of groups

Hyperbolic groups and the more general relatively hyperbolic groups are classes of "negatively curved groups" that are a focal point in geometric group theory. These groups come equipped with a natural boundary at infinity, which is a compact metrizable space on which the group acts. After a brief introduction to hyperbolic and relatively hyperbolic groups, we outline the core ideas behind proving that the orbit equivalence relations of the natural actions of hyperbolic and relatively hyperbolic groups on their boundaries are particularly simple from the point of view of descriptive set theory, namely, that they are hyperfinite.

SAMUEL MELLICK, McGill
Higher rank groups have fixed price one

Cost is a fundamental invariant in measured group theory, generalising the notion of "rank" (in the sense of the minimum number of generators for a group). A group is said to have "fixed price" if all of its actions have the same cost. In recent work, we have been able to show that "higher rank" groups (such as $\mathrm{SL}_3(\mathbb{R})$ and $\mathrm{Aut}(T) \times \mathrm{Aut}(T)$) have fixed price one. This implies, for instance, that lattices in $\mathrm{SL}_3(\mathbb{R})$ admit generating sets of size little-o of their covolume, resolving a conjecture of Abert-Gelander-Nikolov. It also implies state of the art vanishing results for mod-p Betti numbers. A key ingredient in the argument is analysis of a new object from probability theory, the "Ideal Poisson-Voronoi tessellation" (IPVT). In higher rank, this object has truly bizarre properties.
I will give an overview of cost and sketch the structure of the argument. No prior familiarity with cost or the requisite probability theory will be assumed.
Joint work with Mikolaj Fraczyk and Amanda Wilkens.

DIANA CAROLINA MONTOYA, Technische Universität Wien
Maximal independence and singular cardinals

In this talk, we will deal with the concept of a maximal δ-independent family of subsets of λ, when λ is a singular cardinal of cofinality κ and δ is a regular cardinal $\leq \kappa$. We will show that if λ is a singular cardinal which is a limit of a sequence of regular cardinals $(\lambda_\alpha : \alpha < \kappa)$ and there are maximal δ-independent families at each cardinal λ_α; then it is possible to build a maximal δ-independent family at the singular λ. Afterward, we will use this fact together with the results of Kunen regarding the existence of maximal independent families at regular cardinals to prove our main result: If λ is a singular cardinal which is a limit of supercompact cardinals $(\lambda_\alpha : \alpha < \kappa)$ and $cof(\lambda) = \kappa$, then consistently there exists a maximal κ-independent family of subsets of λ. Finally, we add a discussion on the possible sizes of these families.

BRIAN PINSKY, Rutgers University
Groups which are not Automorphism Groups of Graphs

Frucht's theorem states every group is the automrophism group of a graph. This was shown in ZFC in 1960. We show Frucht's theorem also holds in ZF, by a similar proof, but that the proof critically relies on foundation.

In ZFA set theory (ZF with atoms), we will show Frucht's theorem can fail, and there are counterexamples in many common permutation models. Frucht's theorem can also hold in ZFA for non-trivial reasons, as happens in the ordered Mostowski model. We will examine Frucht's theorem over ZFA, and talk about what this might mean for models of ZF.

ANTOINE POULIN, McGill
Borel complexity of Archimedean orders on finitely generated group

We present results on the Borel complexity of the action of $GL_2(\mathbb{Z})$ on the Archimedean orders of $\mathbb{Z}^2$. This mimics a result of F. Calderoni, A. Shani, D. Marker and L. Motto Ros for $\mathbb{Q}^2$. We discuss possible generalizations to different groups, including for intermediate rings $\mathbb{Z} \subset R \subset \mathbb{Q}$ and $\mathbb{Z}^n$.

ASGER TORNQUIST, University of Copenhagen, Denmark
Almost disjoint families in higher dimensions

A classical almost disjoint family is a family of subsets of the natural numbers such that any two non-identical elements of the family intersect finitely, that is, their intersection is in the ideal FIN. A "mad family" is, of course, a maximal almost disjoint family. Definability problems related to classical mad families have been studied intensively in the past few years. This talk is about extending and generalizing the classical notion of an almost disjoint family by replacing the ideal of finite sets FIN with other ideals, and in this talk, this specifically means replacing it with the iterated Frechet ideals FIN^2, FIN^3,... We call mad families with respect to the iterated Frechet ideals "higher dimensional" mad families. In this talk, I will try to give an overview of definability and undefinability results for higiher dimensional mad families. This is joint work with David Schrittesser.

SPENCER UNGER, University of Toronto
Flows on the torus

In this talk, I'll present some of the contents of two separate joint works about flows. First, joint with Andrew Marks, we produce real valued flows between sets whose boundaries have small box dimension which are simpler than the ones from our Borel circle squaring paper. Second, joint with Anton Bernshteyn and Anush Tserunyan, we produce a whole family of flows with different prescribed combinatorial properties. Further we show that our method applies to a class of functions that includes both differences of characteristic functions of sets with small boundary and Holder continuous functions with mean 0.

ALLISON WANG, Carnegie Mellon University
Every CBER is smooth below the Carlson-Simpson generic partition

One difficulty that arises in studying the class of countable Borel equivalence relations (CBERs) is that in many cases, the complexity of a CBER lies on a "small" set. For instance, a result of Hjorth and Kechris states that every CBER on a Polish space is hyperfinite when restricted to some comeager set. Another result, due to Mathias, shows that every CBER on the Ellentuck Ramsey space is hyperfinite when restricted to some pure Ellentuck cube. In this talk, we will show that every CBER on the space of all infinite partitions of the natural numbers coincides with equality below a Carlson-Simpson generic element. This is joint work with Aristotelis Panagiotopoulos.