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Green's functions for RM points.

Let τ_1 and τ_2 be real quadratic elements of discriminants D_1 and D_2 ($D_1 \neq D_2$) of the Drinfeld p-adic upper half plane, let τ_j' be the conjugate of τ_j, let $\Gamma_j \subset \mathbf{SL_2}(\mathbb{Z})$ be the stabiliser of τ_j, and let $(\tau_1, \tau_1') \cdot (\tau_2, \tau_2') \in \{-1, 0, 1\}$ be the topological intersection on the Poincare upper half plane of the two hyperbolic geodesics with these endpoints. Let M_n be the set of 2×2 matrices with integer entries with determinant p. Then the quantity

$$G_n(\tau_1, \tau_2) = \left(\prod_{\gamma \in \Gamma_1 \backslash M_n / \Gamma_2} g(\tau_1, \gamma\tau_2)^{(\tau_1, \tau_1') \cdot \gamma(\tau_2, \tau_2')} \right)^{12}, \quad \text{where} \quad g(z_1, z_2) := \frac{(z_1 - z_2)(z_1' - z_2')}{(z_1 - z_1')(z_2 - z_2')}$$

converges to an element of $\mathbb{Q}_p$ as n tends to infinity. When $p = 2, 3, 5, 7,$ or 13, this quantity is expected to be algebraic and to belong to the compositum of the narrow Hilbert class fields of $\mathbb{Q}(\sqrt{D_1})$ and $\mathbb{Q}(\sqrt{D_2})$. For other p it is expected to be transcendental in general. I will describe a conceptual framework for understanding these assertion by interpreting $G(\tau_1, \tau_2) := \lim_{n \to \infty} G_n(\tau_1, \tau_2)$ as the value of an (exponential) Green's function at the pair (τ_1, τ_2) of RM points.

This is a report on a piece of an ongoing joint project with Jan Vonk.