Recent Advances in Mathematical Finance
Progrès récents en finance mathématique

(Org: **Alexandru Badescu** (University of Calgary) and/et **Cody Hyndman** (Concordia University))

LOUIS ARSENAULT-MAHJOUBI, Simon Fraser University
Discrete nonlinear filtering in finance: Applications to stochastic volatility models with jumps

The estimation of complex financial models such as jump-diffusion models is often performed using sophisticated filtering methods. The discrete nonlinear filter (DNF) provides a deterministic, quick, and flexible alternative to the popular particle filter. Recently, researchers have developed a high-dimensional version of the DNF for jump-diffusion models. In this talk, I outline how the filter is applicable for both frequentist and Bayesian estimation, areas where it is particularly effective compared to alternative approaches (e.g., joint estimation with options and returns data), and its limitations (e.g., models with multiple persistent latent variables). Moreover, I will provide evidence of its effectiveness from simulation studies, empirical results from S&P 500 returns, and a brief overview of the SVDNF R package, which makes the DNF available for returns-only estimation of one-factor stochastic volatility models.

MACIEJ AUGUSTYNIAK, University of Montreal
A Discrete-Time Hedging Framework for Econometric Option Pricing Models

We present a quadratic hedging framework for a general class of discrete-time affine multi-factor models. A semi-explicit hedging formula is derived for our general framework which applies to a myriad of the option pricing models proposed in the discrete-time literature, including the multi-component fat-tailed GARCH model, the Lévy GARCH model, the affine realized volatility (GARV) model, and the heterogeneous autoregressive gamma (HARG) model for realized volatility. Additionally, we conduct an extensive empirical study of the impact of modelling features on the hedging effectiveness of S&P 500 options. Overall, we find that fat tails can be credited for half of the hedging improvement observed, while a second volatility factor and a non-monotonic pricing kernel each contribute to a quarter of this improvement. Interestingly, our study indicates that the added value of these features for hedging is different than for pricing. A robustness analysis shows that a similar conclusion can be reached when considering the Dow Jones Industrial Average. The talk will also cover some extensions of our methodology that incorporate stochastic interest rates and basis risk.

JEAN-FRANÇOIS BÉGIN, Simon Fraser University
A general option pricing framework for affine fractionally integrated models

This article studies the impact of fractional integration on volatility modelling and option pricing. We propose a general discrete-time pricing framework based on affine multi-component volatility models that admit ARCH(∞) representations. This not only nests a large variety of option pricing models from the literature, but also allows for the introduction of novel covariance-stationary long-memory affine GARCH pricing models. Using an infinite sum characterization of the log-asset price's cumulant generating function, we derive semi-explicit expressions for the valuation of European-style derivatives under a general variance-dependent stochastic discount factor. Moreover, we carry out an extensive empirical analysis using returns and S&P 500 options over the period 1996–2019. Overall, we find that once the informational content from options is incorporated into the parameter estimation process, the inclusion of fractionally integrated dynamics in volatility is beneficial for improving the out-of-sample option pricing performance. The largest improvements in the implied volatility root-mean-square errors occur for options with maturities longer than one year, reaching 33% and 13% when compared to standard one- and two-component short-memory models, respectively.

ZITENG CHENG, University of Toronto
Mean field regrets in discrete time games

We use mean field games (MFGs) to investigate approximations of N-player games with uniformly symmetrically continuous heterogeneous closed-loop actions. To incorporate agents' risk aversion (beyond the classical expected total costs), we use an abstract evaluation functional for their performance criteria. Centered around the notion of regret, we conduct non-asymptotic analysis on the approximation capability of MFGs from the perspective of state-action distribution without requiring the uniqueness of equilibria. Under suitable assumptions, we first show that scenarios in the N-player games with large N and small average regrets can be well approximated by approximate solutions of MFGs with relatively small regrets. We then show that δ-mean field equilibria can be used to construct ε-equilibria in N-player games. Furthermore, in this general setting, we prove the existence of mean field equilibria.

DENA FIROOZI, HEC Montréal - Université de Montréal
LQG Risk-Sensitive Mean Field Games with a Major Agent

Risk sensitivity plays an important role in the study of finance and economics as risk-neutral models cannot capture and justify all economic behaviors observed in reality. Risk-sensitive mean field game theory was developed recently for systems where there exists a large number of indistinguishable, asymptotically negligible and heterogeneous risk-sensitive players, who are coupled via the empirical distribution of state across population (average state of the population in the LQG case). In this work, we extend the theory of LQG risk-sensitive MFGs to the setup where there exists one major agent as well as a large number of minor agents. The major agent has a significant impact on each minor agent and its impact does not collapse with the increase in the number of minor agents. Each agent is subject to linear dynamics with an exponential-of-integral quadratic cost functional. Moreover, all agents interact via the average state of minor agents (so-called empirical mean field) and the major agent's state. We use a change of measure technique together with a variational analysis to derive the best response strategies of agents in the limiting case where the number of agents goes to infinity. We establish that the set of obtained best-response strategies yields a Nash equilibrium in the limiting case and an ϵ -Nash equilibrium in the finite player case.

CHRISTOPH FREI, University of Alberta
Principal Trading Arrangements: Optimality under Temporary and Permanent Price Impact

We study the optimal execution problem in a principal-agent setting. A client (for example, a pension fund, endowment, or other institution) contracts to purchase a large position from a dealer at a future point in time. In the interim, the dealer acquires the position from the market, choosing how to divide his trading across time. Price impact may have temporary and permanent components. There is hidden action in that the client cannot directly dictate the dealer's trades. Rather, she chooses a contract with the goal of minimizing her expected payment, given the price process and an understanding of the dealer's incentives. Many contracts used in practice prescribe a payment equal to some weighted average of the market prices within the execution window. We explicitly characterize the optimal such weights: they are symmetric and generally U-shaped over time. The talk is based on joint work with Markus Baldauf (University of British Columbia) and Joshua Mollner (Northwestern University).

FRÉDÉRIC GODIN, Concordia University
Risk allocation through Shapley decompositions with applications to variable annuities

A flexible risk decomposition method for life insurance contracts embedding several risk factors is introduced. Hedging can be naturally embedded in the framework. Although the method is applied to variable annuities in this work, it is also applicable in general to other insurance or financial contracts. The approach relies on applying an allocation principle to components of a Shapley decomposition of the gain and loss. The implementation of the allocation method requires a stochastic on stochastic algorithm. Numerical examples studying the relative impact of equity, interest rate and mortality risk for guaranteed minimal maturity benefit (GMMB) policies conclude our analysis.

KIRILL GOLUBNICHIY, University of Calgary
Ill-Posed Problem for the Black-Scholes Equation solution and Machine Learning

In the previous paper (Inverse Problems, 32, 015010, 2016), a new heuristic mathematical model was proposed for accurate forecasting of prices of stock options for 1-2 trading days ahead of the present one. This new technique uses the Black-Scholes equation supplied by new intervals for the underlying stock and new initial and boundary conditions for option prices. The Black-Scholes equation was solved in the positive direction of the time variable, This ill-posed initial boundary value problem was solved by the so-called Quasi-Reversibility Method (QRM). This approach with an added trading strategy was tested on the market data for 368 stock options and good forecasting results were demonstrated. We use the geometric Brownian motion to provide an explanation of that effectivity using computationally simulated data for European call options. We also provide a convergence analysis for QRM. The key tool of that analysis is a Carleman estimate. To enhance these results, the Neural Network Machine Learning is applied on the second stage. Real market data are used. Results of Quasi-Reversibility Method and Machine Learning method are compared in terms of accuracy, precision and recall.

MATHEUS GRASSELLI, McMaster University
Green monetary policy

It is well documented that climate change adaptation and mitigation, including the transition to net zero emissions, require financial flows that are several times larger than what is currently observed. In this talk, I review a stock-flow consistent climate-economy model and explain the inherent instability associated with balancing the effects of economic damages caused by climate change and the financial burden incurred to prevent them. I then present a modification of the model that takes into account two different monetary policies that could be implemented by central banks in order to improve the stability of the system: green quantitative easing (that is to say, large scale purchases of financial instruments used to fund green investment) and green capital requirements (that is, measures aimed at making loans for green projects more attractive for banks to hold in their balance sheets). I illustrate the stabilization effects of these polices with examples calibrated to data and similar proposals in the literature.

ANASTASIS KRATSIOS, McMaster
Designing Universal Causal Deep Learning Models: The Geometric (Hyper)Transformer

Several problems in stochastic analysis are defined through their geometry, and preserving that geometric structure is essential to generating meaningful predictions. Nevertheless, how to design principled deep learning (DL) models capable of encoding these geometric structures remains largely unknown. We address this open problem by introducing a universal causal geometric DL framework in which the user specifies a suitable pair of metric spaces X and Y and our framework returns a DL model capable of causally approximating any "regular" map sending time series in XZ to time series in YZ while respecting their forward flow of information throughout time. Suitable geometries on Y include various (adapted) Wasserstein spaces arising in optimal stopping problems, a variety of statistical manifolds describing the conditional distribution of continuous-time finite state Markov chains, and all Fréchet spaces admitting a Schauder basis, e.g. as in classical finance. Suitable spaces X are compact subsets of any Euclidean space. Our results all quantitatively express the number of parameters needed for our DL model to achieve a given approximation error as a function of the target map's regularity and the geometric structure both of X and of Y. Even when omitting any temporal structure, our universal approximation theorems are the first guarantees that Hölder functions, defined between such X and Y can be approximated by DL models.

Joint work with: Beatrice Acciaio, Gudmund Pammer

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LUCA LALOR, University of Calgary
A Numerical Solution to an Algorithmic and HFT Problem with a Jump-Diffusion Price Process

The main subject of this talk is to introduce an algorithmic and High-Frequency Trading model where the price process is of the jump-diffusion type. This talk begins with a brief introduction on how to apply Stochastic Optimal Control theory to algorithmic trading problems. A price process in the Jump-Diffusion setting is then introduced along with its infinitesimal generator, which encompasses one of the major modelling adjustments in this research. Previous research modelled the jumps through a diffusion approximation, while here the jumps are modelled directly. Preliminary results, using an Implicit-Explicit Finite Difference Scheme, for an Optimal Acquisition algorithmic trading problem will be presented. Here the jump part of the Jump-Diffusion price process will be a function of a Poisson process. This talk will end with a discussion on proposed modifications to the discussed algorithmic trading problem, so that the future models will account for the non-Markovian property seen in LOB data.

ANNE MACKAY, Université de Sherbrooke
Optimal stopping with a discontinuous and time-dependent reward function

We consider a financial derivative with early exercise whose reward function is time-dependent, unbounded and presents a discontinuity at maturity. In this context, the regularity conditions required to apply the results and techniques used in the American option literature are not satisfied. We confirm the existence of an optimal stopping time, and show that our problem admits a trivial optimal stopping time under certain conditions. We show that the value function admits another representation in terms of a continuous reward function, which allows us to express the price of our derivative as a free boundary value problem. We also present an integral expression for the early exercise premium and an application of our results to the problem of optimal surrender in variable annuities.

ROMAN MAKAROV, Wilfrid Laurier University
Structural Credit Risk Models with Occupation Times and Spectral Expansions

We propose a class of structural credit risk models with liquidation barriers and hazard rates based on occupation times. The defaults within the models are characterized in accordance with Chapter 7 (a liquidation process) and Chapter 11 (a reorganization process) of the U.S. Bankruptcy Code. The risk-neutral default probabilities involve joint probability distributions of the underlying firm's value with imposed killing at the liquidation barrier and its occupation time with respect to the reorganization barrier. The joint probability distributions are expressed as spectral series expansions, which allow us to write pricing formulas for credit derivatives and credit default swap (CDS) spreads explicitly as infinite series that converge rapidly. The spectral methodology works for solvable diffusion, such as the geometric Brownian motion (GBM), the constant elasticity of variance (CEV) process and other state-dependent volatility diffusion models. We then calibrated our model with a GBM governing the firm's value to market CDS spreads from the Total Energy company. Our calibration results show that the computations are fast, and the fit is nearly perfect.

This is a joint work with Giuseppe Campolieti and Hiromichi Kato.

ADAM METZLER, Wilfrid Laurier University
(Machine) Learning From Transaction-Level Investment Account Data

In this talk I will give a high-level overview of several recent projects related to a rich set of transaction-level data from individual investment accounts. Time permitting I will touch on (i) clustering individuals based on their trading behaviour, (ii) assessing the degree to which client behaviour aligns with professional advice, (iii) understanding the relationship between savings habits and investment outcomes and (iv) the extent to which professional advice can be replicated by classification algorithms.

MARK REESOR, Wilfrid Laurier Universtiy
Incorporating Climate Risk into Portfolio Credit Risk Models via Distortion

Regulatory requirements are evolving towards mandating financial institutions to estimate and report their climate-related financial risks. Climate risks classify into two broad categories — physical and transition — and, being medium- to long-term in nature, they are important risk factors for credit portfolios. Threshold models for portfolio credit risk specify account level models with both systematic and idiosyncratic effects. These aggregate to generate the portfolio loss distribution from which risk metrics are calculated. Augmenting the systematic factor with climate factors is one method to incorporate climate risk into existing models. Distortion provides a method for re-weighting a probability distribution. The amount of deformation depends on the choice of distortion function and its parameter. Here, we propose distortion as a way of incorporating climate risk into existing credit risk models. Some properties of the distorted credit risk models are derived and explored. The connection between distortion functions and constrained relative entropy optimisation provides insight into distortion function structure and parameter values. This is joint work with Arie Zeldenrijk, Mark Drmac, and Walid Mnif.

JEAN-FRANÇOIS RENAUD, UQAM

Maximization of dividend payments with a concave bound on the dividend rate

In a Brownian model, we revisit Bruno de Finetti's optimal dividends problem for absolutely continuous strategies by imposing a path-dependent concave bound on the rate of dividend payments. This is a generalization of the classical version of this problem as studied by Jeanblanc and Shiryaev (1995) and in which the rate is uniformly bounded. Our main result consists in proving that a so-called mean-reverting strategy is optimal. Then, we consider the associated bail-out optimization problem in which the cash process must be kept solvent. Again, we obtain that it is optimal to pay out dividends using a mean-reverting strategy, while bail-out payments are made to avoid bankruptcy.

This talk is based on joint papers with Félix Locas, Alexandre Roch and Clarence Simard.

ALEXANDRE ROCH, UQAM

Optimal dividend and capital injection strategies : a viscosity approach

We consider the problem of finding the optimal dividend and capital injection strategies for a firm for which the cash (surplus) process is driven by Brownian motion. Dividend processes are assumed to be absolutely continuous whereas capital injections are only assumed to be adapted and non decreasing. We show that when it is only optimal to inject capital, we only do so to avoid bankruptcy. Using the theory of viscosity solutions and dynamic programming, we show that the value function is the unique viscosity solution of an associated HJB variational inequality. We characterize the optimal dividend strategy in terms of a threshold solution. We prove a comparison theorem for viscosity solutions which is used to show that the optimal solution to the problem is dichotomic : either we inject capital to avoid bankruptcy in the minimal way, or we never inject and let the firm default when the cash process hits zero.

This talk is based on joint work with and builds on previous talks by Jean-François Renaud and Clarence Simard.

DAVID SAUNDERS, University of Waterloo

Bounds on Choquet Integrals in Finite Product Spaces for Capacities with Given Marginals

We investigate the problem of finding upper and lower bounds for a Choquet risk measure of a nonlinear function of two risk factors, when the marginal distributions of the risk factors are ambiguous and represented by nonadditive measures on the marginal spaces, but the joint nonadditive distribution on the product space is unknown. We treat this problem as a generalization of the optimal transportation problem to the setting of nonadditive measures. We provide explicit characterizations of the optimal solutions for finite marginal spaces, and we investigate some of their properties. We further discuss the connections with linear programming, showing that the optimal transport problems for capacities are linear programs, and we also characterize their duals explicitly.

CLARENCE SIMARD, Université du Québec à Montréal

Optimal dividend with a proportional bound in a Brownian model

We study the problem of dividend optimization when the value of the firm is a Brownian motion with drift and the rate of dividend is bounded by a percentage of the firm's value. We will see that the optimal dividend strategy is a bridge between the optimal strategy when dividends are unbounded, derived in Jeanblanc and Shiryaev (1995), and the mean-reverting strategy studied in Avanzi and Wong (2012).

ANTONY WARE, University of Calgary
Multi-factor polynomial models for energy commodity markets

In the context of energy commodity price modelling, prices are often formed from exponential maps of underlying factor processes, motivated in part by the mathematical convenience this offers. In this talk we will focus on multi-factor models based on polynomial maps of polynomial processes (PMPP models), and explore how they can function in a similar way, retaining much of the mathematical convenience of the exponential map, and providing additional flexibility, such as the ability to capture negative prices in a natural way, or to model prices with intrinsic upper bounds. And this additional level of flexibility means that PMPP models are capable of capturing the extreme dynamics that are commonly seen in energy market prices even with relatively tame dynamics in the underlying factor process.

Polynomial processes have the property that the expectation of a polynomial map of the process value at a future time T, conditional on its value at an earlier time t, is also a polynomial map of the same (or lower) degree. In the context of PMPP models, this property means that futures prices can be computed via multiplication by a (typically small) matrix. We will demonstrate how this works in practice, and show how option prices can also be computed in semi-closed form, using techniques that open the door to other applications.

FRANÇOIS WATIER, Université du Québec à Montréal
A Weighted Mean-Variance Portfolio Under a No-Bankruptcy Constraint

In the context of a continuous-time Black-Scholes market model, we consider a stochastic portfolio management problem where the investor wishes to reach an expected terminal wealth while minimizing a skewed mean-variance risk measure, that is more weight (or penalty) is given on downside returns. Under a positive wealth constraint, we show that there exists an explicit optimal strategy given in feedback form.

Joint work with Rene Ferland