Noncommutative Geometry and Mathematical Physics
Géométrie non commutative et physique mathématique
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HEATH EMERSON, University of Victoria
Heisenberg spectral cycles for flows

The annihilation and creation operators x+d/dx, x-d/dx, assemble to an even spectral triple over the crossed product of the C*-algebra A of bounded uniformly continuous functions on the real line, by the real line as a discrete topological group. Taking a single orbit of a smooth (ergodic) flow on a compact manifold determines, by restricting functions to the orbit, a C*-subalgebra of A and, taking into account the spectral triple, one gets an example of a 'Noncommutative Geometry' in the sense of A. Connes, associated to the flow. I will discuss application of the Local Index Formula to this situation, with especial attention to the analytic zeta function associated to the spectral triple. The meromorphic extension problem and pole structure of this zeta function turns out to be related to certain delicate points in the study of ergodic time averages. Krönecker flow on the 2-torus already illustrates this in an interesting way, as we will briefly explain.

MICHAEL FRANCIS, University of Western Ontario
Homological unitality of smooth groupoid algebras

For any Lie group G, the smooth convolution algebra $C_c^\infty(G)$ is nonunital (unless G is discrete), but a celebrated result of Dixmier-Malliavin says the following weaker property holds:

$$\text{Every } \varphi \in C_c^\infty(G) \text{ is a finite sum } \sum f_i * g_i \text{ where } f_i, g_i \in C_c^\infty(G).$$

In a recent article, I extended this result to the case where G is a Lie groupoid. Writing $A = C_c^\infty(G)$, this says exactly that the map $A \otimes A \to A$ defined by convolution product is surjective. Continuing this work, I show that A is homologically unital in the sense of Wodzicki, meaning the bar complex

$$\cdots \longrightarrow A^{\otimes 4} \longrightarrow A^{\otimes 3} \longrightarrow A^{\otimes 2} \longrightarrow A \longrightarrow 0$$

is exact. Wodzikci showed homological unitality is precisely the property needed by an ideal to perform excision in cyclic/Hochschild homology, i.e. the condition for a short exact sequence of algebras to induce a long exact sequence.

For a Lie groupoid G with base X, the concept of an invariant submanifold $Y \subseteq X$ is meaningful (this is consistent with the usual meaning in the group action case). In terms of the smooth convolution algebra, invariant submanifolds manifest as ideals $I_Y^k \subseteq C_c^\infty(G)$, where k encodes an order of vanishing along Y. I furthermore show that I_Y^k is homologically unital for $k = \infty$, which means excision holds for infinite-order vanishing ideals associated to invariant submanifolds. This result gives an organizing principle for calculating cyclic/Hochschild homology: localize the calculation around invariant submanifolds.

SITANSHU GAKKHAR, California Institute of Technology
A quantum stochastic approach to spectral action

We review the spectral action for Robertson-Walker cosmologies and consider an approach through quantum stochastic processes. Towards this, we study the heat semigroups for almost commutative spectral triples characterized as endomorphim subalgebras of spinor bundles, and show that they are quantum dynamical semigroups. Then using the Goswami-Sinha quantum stochastic calculus, the existence of Evans-Hudson flows which can be viewed as defining diffusion processes on the spectral triple is established.

DANIEL HUDSON, University of Toronto
Weightings for Lie Groupoids and Lie Algebroids

Weightings, or quasi-homogeneous structures, are a concept introduced by Melrose and later by Loizides and Meinrenken as a way to generalize the notion of order of vanishing. Loizides and Meinreken show that, given a weighting, one can define a 'weighted normal bundle' and a 'weighted deformation space' in a way that generalizes the standard normal bundle and standard deformation to the normal cone. In this talk, we will discuss the appropriate notions of weightings in the case of Lie algebroids and Lie groupoids so as to ensure that a number of desirable properties hold.

THERESE LANDRY, University of California, Santa Barbara
Spectral Triples for Noncommutative Solenoids

We construct odd finitely summable spectral triples based on length functions of bounded doubling on noncommutative solenoids. Our spectral triples induce a Leibniz Lip– norm on the state spaces of the noncommutative solenoids, giving them the structure of Leibniz quantum compact metric spaces. By applying methods of R. Floricel and A. Ghorbanpour, we also show that our odd spectral triples on noncommutative solenoids can be considered as direct limits of spectral triples on rotation algebras.

YIANNIS LOIZIDES, George Mason University
A fixed-point formula for Dirac operators on Lie groupoids

I will describe an equivariant index formula for a family of Dirac operators on the source fibres of a Lie groupoid. The result complements work of Heitsch-Lazarov and Pflaum-Posthuma-Tang. This is joint work with Liu, Sadegh and Sanchez. This will be one part of a two-part talk, the other part being the talk of A.R.H.S. Sadegh.

EDWARD MCDONALD, Penn State University
The Dixmier trace and the Density of States

The Dixmier trace has been employed by Connes for several purposes, including defining the integral in noncommutative geometry. Connes' integration formula can be viewed as a way of associating a measure to a self-adjoint operator. In solid state physics there is another celebrated measure associated with Schrodinger operators: the density of states. Using techniques from noncommutative geometry, we have recently proved that the density of states can in many cases be computed by a Dixmier trace. This work also provides a new perspective on Roe's index theorem for open manifolds by giving a Dixmier trace formula for the index. Joint work with N. Azamov, E. Hekkelman, F. Sukochev and D. Zanin.

NATHAN PAGLIAROLI, Western University
Liouville Quantum gravity from Noncommutative Geometry

In this talk we will highlight recent developments in toy models of Quantum Gravity originating from Noncommutative Geometry. The models of interest are finite real spectral triples equipped with a path integral over the space of possible Dirac operators, dubbed Dirac ensembles. In the noncommutative geometric setting of spectral triples, Dirac operators take the center stage as a replacement for a metric on a manifold. Thus, this path integral serves as a noncommutative analogue of integration over metrics, a key feature of a theory of quantum gravity.

Such models can be shown to be bi-tracial random matrix integrals. Using well-established rigorous techniques of Random Matrix Theory, we derive the critical exponents and the asymptotic expansion of partition functions of various Dirac ensembles which match that of minimal models from Liouville conformal field theory coupled with gravity.

AHMAD REZA HAJ SAEEDI SADEGH, Northeastern University
Deformation Spaces and localized equivariant index formulas for groupoids

This talk is based on joint work with Liu, Sanchez, and Loizides. In this talk, we discuss the application of deformation spaces in studying localized equivariant index formulas for groupoids. This work generalizes the results of Higson-Yi and Braverman-Haj. This is part of a two-part talk, the other delivered by Yiannis Loizides.

JESUS SANCHEZ JR., Washington University in St Louis
The Spectral Zeta Cocycle

In this talk we use D. Quillen's Chern-Weil theory for cyclic cohomology to calculate the transgression relations for the spectral zeta cocycle introduced by N. Higson in relation to the Connes-Moscovici noncommutative local index theorem. We use the transgression relations to give an explicit homotopy to the Connes-Chern character.

ILYA SHAPIRO, University of Windsor
Hopf-cyclic coefficients for a Hopf algebra in a rigid braided category.

A classical anti-Yetter-Drinfeld module for a Hopf algebra H was defined by Hajac-Khalkhali-Rangipour-Sommerhauser as a module and a comodule over H such that the two structures are compatible in a specific sense. These objects serve as necessary coefficients for cyclic cohomology theories of H-equivariant algebras.

If $\mathcal{B}$ is a braided category then there is a notion of a Hopf algebra H in $\mathcal{B}$. A braided version of anti-Yetter-Drinfeld modules has been considered by Khalkhali-Pourkia, and more recently by Bartulovic. These approaches generalize the classical definition, and are successful to the point of 1-dimensional coefficients (modular pairs in involution) for a balanced braided $\mathcal{B}$.

On the other hand, the classical definition has been variously generalized, and in particular, it is now possible to talk about anti-Yetter-Drinfeld modules for a monoidal category. Note that H-modules in $\mathcal{B}$ form a monoidal category $\mathcal{C}$. We will describe anti-Yetter-Drinfeld modules for $\mathcal{C}$ as modules and comodules (compatibly) over H, but not, as one would guess, in $\mathcal{B}$. Instead, one needs to replace $\mathcal{B}$ with anti-Yetter-Drinfeld modules for $\mathcal{B}$. This leads to an interesting decomposition result for the category of coefficients. The 1-dimensional coefficients for a balanced braided $\mathcal{B}$, mentioned above, form a part of this decomposition, when they exist.

DAMIEN TAGEDDINE, McGill University
Noncommutative geometry on discrete spaces

The approximation theory of partial differential equations on smooth manifolds can take several aspects. The various methods rely on the intuitive geometric idea that the fine structure of space is discrete. The resulting discretized space is governed by a small parameter $\hbar$ which plays the role of infinitesimal.

A natural question is to determine a general framework, i.e. a unifying treatment of continuous and discrete geometries. Noncommutative differential geometry, and more specifically the construction of spectral triples, offers a promising direction to formalize approximation theory of differential operator using the same tool as classical differential geometry.

In this talk, we will start by reviewing how to associate a spectral triple to a triangulation of a given smooth manifold. Then, we will show how a sequence of spectral triples, associated to a sequence of refined triangulations, can be related at the limit $\hbar \to 0$ to the spectral triple on a spin manifold. In fact, we observe a convergence in average when the Dirac operators are taken as random matrices with a suitable distribution on the coefficients.

VENKATA KARTHIK TIMMAVAJJULA, University of New Brunswick, Fredericton
Extended diffeomorphism groups for noncommutative manifolds

Given an unital pre-C^* algebra B with a $*$-exterior algebra, one can define extended diffeomorphism group for the noncommutative manifold B and its subgroup of topologically trivial elements. Using results of Bratteli–Elliott–Jorgensen and Krähmer, we compute these groups when B is an irrational noncommutative 2-torus and the algebraic standard Podleś sphere, respectively. We then apply this to the computation of moduli spaces of solutions to Euclidean Maxwell's equations with fixed topological sector and current 1-form.

This is joint work with B. Ćaćić.

LUUK VERHOEVEN, UWO

Fermionic fuzzy geometries

A random fuzzy geometry consists of a fuzzy geometry, i.e. a spectral triple $(M_N(\mathbb{C}), V \otimes M_N(\mathbb{C}), D)$ where the Dirac operator is a random variable with some predetermined distribution of the form $\frac{1}{Z}e^{-S(D)}dD$, traditionally these actions $S(D)$ consist of traces of powers of D. In this talk I will discuss some of the difficulties associated to adding a fermionic term to this action as well as some new results on the effect of such a fermionic term for a fuzzy geometry of signature $(0, 1)$.

This is joint work with Nathan Pagliaroli and Masoud Khalkhali.