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Variational properties of square root LASSO: Smoothness, uniqueness, explicit solutions

The square root LASSO (SR-LASSO) is a powerful sparse regularization technique widely adopted in statistics, and increasingly popular in the scientific computing and machine learning communities. SR-LASSO is the sum of a data fidelity term and a one-norm weighted by a tuning parameter. It closely resembles the unconstrained formulation of LASSO (Least Absolute Shrinkage and Selection Operator), essentially obtained by "removing" the square from the latter's data fidelity term. This algebraic transformation corresponds with optimal tuning strategies for SR-LASSO that are robust to unknown observation errors. Our goal is to study the (generally, set-valued) solution map of SR-LASSO to determine its sensitivity to the measurement vector and tuning parameter. We present how three increasingly strict assumptions give rise to correspondingly "nice" properties of the SR-LASSO solution map. Our investigation is based on variational analysis, continuing a line of work initiated by the current authors for unconstrained LASSO. First, we show the weakest assumption yields uniqueness for SR-LASSO solutions. The intermediate assumption additionally yields directional differentiability (hence Lipschitzness) of the solution map, as well as an analytic expression for the solution. The final assumption yields continuous differentiability (with respect to the measurement vector and tuning parameter). When the solution is Lipschitz we obtain informative explicit bounds on the Lipschitz constant and contrast this quantification of sensitivity with that for unconstrained LASSO. Compelling numerics flesh out the theoretical discussion.