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Generalized pseudo-Anosov Maps and Hubbard Trees

The Nielsen-Thurston classification of the mapping classes proved that every orientation preserving homeomorphism of a closed surface, up to isotopy is either periodic, reducible, or pseudo-Anosov. Pseudo-Anosov maps have particularly nice structure because they expand along one foliation by a factor of $\lambda > 1$ and contract along a transversal foliation by a factor of $\frac{1}{\lambda}$. The number λ is called the dilatation of the pseudo-Anosov. Thurston showed that every dilatation λ of a pseudo-Anosov map is an algebraic unit, and conjectured that every algebraic unit λ whose Galois conjugates lie in the annulus $A_\lambda = \{z : \frac{1}{\lambda} < |z| < \lambda\}$ is a dilatation of some pseudo-Anosov on some surface S.

Pseudo-Anosovs have a huge role in Teichmuller theory and geometric topology. The relation between these and complex dynamics has been well studied inspired by Thurston.

In this project, I develop a new connection between the dynamics of quadratic polynomials on the complex plane and the dynamics of homeomorphisms of surfaces. In particular, given a quadratic polynomial, we show that one can construct an extension of it which is generalized pseudo-Anosov homeomorphism. Generalized pseudo-Anosov means the foliations have infinite singularities that accumulate on finitely many points. We determine for which quadratic polynomials such an extension exists. My construction is related to the dynamics on the Hubbard tree which is a forward invariant subset of the filled Julia set that contains the critical orbit.