Geometry for Partial Differential Equations
Géométrie pour les équations différentielles partielles

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PAULA BURKHARDT-GUIM, NYU Courant
ADM mass for C^0 metrics and distortion under Ricci-DeTurck flow

We show that there exists a quantity, depending only on C^0 data of a Riemannian metric, that agrees with the usual ADM mass at infinity whenever the ADM mass exists, but has a well-defined limit at infinity for any continuous Riemannian metric that is asymptotically flat in the C^0 sense and has nonnegative scalar curvature in the sense of Ricci flow. Moreover, the C^0 mass at infinity is independent of choice of C^0-asymptotically flat coordinate chart, and the C^0 local mass has controlled distortion under Ricci-DeTurck flow when coupled with a suitably evolving test function.

GOONG CHEN, Mathematics Department, Texas A&M University
Animal Motions and Their Fourier Decomposition

in this talk, we first compute the fundamental modes of motion of certain animals. We then computationally analyze their motion by performing Fourier transforms on their trajectory. Their frequencies show strong consistency with the computed frequencies of the fundamental modes of motion. Concrete examples will be illustrated.

SHAOHUA CHEN, Cape Breton University
Global solutions for the 1-D compressible Euler equations with time-dependent damping

In this talk we discuss the Cauchy problem for the 1-D compressible Euler equations with time-dependent damping. We prove the existence of global solutions under the assumptions that the derivatives of initial data are suitable small and the initial volume is large without the condition of small perturbations to the constant initial data. Our approach is based on estimates of the derivatives of Riemann invariants along two characteristic curves.

PARISA FATHEDDIN, Ohio State University, Marion
Asymptotic Behavior of Stochastic Navier-Stokes and Schrodinger Equations

We will consider the stochastic Navier-Stokes and stochastic Schrodinger equations and discuss their asymptotic limits such as large and moderate deviations, central limit theorem and the law of the iterated logarithm. To achieve the large deviation principle, we apply both techniques available in the literature: Azencott method and the weak convergence approach and compare the two methods. The Azencott method is then used to derive the law of the iterated logarithm. Also I will discuss my recently published book for graduate students: "Teaching and Research in Mathematics: A Guide with applications to Industry".

HAIFENG HU, McGill University
Structural Stability for 1D Semiconductor Hydrodynamic Model with Sonic Boundary

In this talk, I will present our recent research on the structural stability of interior subsonic steady states to the hydrodynamic model for semiconductors with sonic boundary. More precisely, we show that the small perturbation in the subsonic doping profiles leads to the small difference between the corresponding interior subsonic solutions. While it has been proved that this model possesses various physical steady states such as the interior subsonic, interior supersonic, shock transonic and smooth transonic solutions, the singularities at the sonic boundary make it difficult to investigate the structural stability of these solutions. To address this issue, we propose a novel approach, which combines the weighted multiplier technique, local singularity analysis, monotonicity argument and squeezing skill. Our work indicates that the interior subsonic solutions are at least amenable to this approach. Numerical approximations further confirm our theoretical results. These results to some extent provide insights into the structural stability of other types of solutions. This is the joint work with Yuehong Feng and Ming Mei.

TUOXIN LI, University of British Columbia
Beckner's inequality for axially symmetric functions on $\mathbb{S}^4$ and $\mathbb{S}^6$

In this talk, we will show that the solutions for a Q-curvature type equation with a Paneitz operator on $\mathbb{S}^n$ in axially symmetric function spaces are constants when n=4, 6. As a result, we establish sharp Beckner's inequalities, which can be viewed as a higher order Moser-Trudinger-Onofri type inequality. We first reduce the equation to 1 dimension by axially symmetry and then study the coefficient of the solution in a suitable orthogonal expansion. I will also introduce some related open problems and point out the difficulties. It is a joint work with Juncheng Wei and Zikai Ye.

XINYANG LU, Lakehead University
Regularity of equations from epitaxial growth

Epitaxial growth is a process where a thin film of material is deposited onto a much thicker substrate. It is currently widely used in precision manufacturing, due to its ability to produce high quality crystals. Since film and substrate are generally made of different materials, there is a mismatch between their material coefficients, hence the optimal distributions are generally non uniform. Their evolution is often modelled by high, e.g. fourth or even sixth, order PDEs. Analyzing their solutions is thus challenging. In this talk we will present recent results on solutions of several PDEs arising from expitaxial growth.

JEROME QUINTIN, University of Waterloo
Toward a non-perturbative understanding of a non-singular universe

The field of numerical relativity has grown tremendously over recent years and led to impressive results, especially in simulations of black hole mergers, which are now observed through gravitational waves. There is more and more interest in using numerical relativity to test modifications to general relativity that would represent new physics in strong gravity regimes, such as around black holes or in the early universe. For such new theories, the challenge is often finding a well-posed formulation of the set of partial differential equations that govern spacetime geometry. We are tackling this problem for a theory called the Cuscuton, a modified gravity theory that, among other things, admits perturbatively stable non-singular cosmological solutions. To test the solution in full (i.e., non-perturbatively) will require numerical relativity techniques. In this talk, I present our recent progress in this direction.

CHONG WANG, Washington and Lee University
Periodic Minimizers of A Ternary Nonlocal Isoperimetric Problem

We study a two-dimensional ternary inhibitory system. The free energy functional combines an interface energy favoring micro-domain growth with a Coulomb-type long range interaction energy which prevents micro-domains from unlimited spreading. Here we consider a limit in which two species are vanishingly small, but interactions are correspondingly large to maintain a nontrivial limit. In this limit two energy levels are distinguished: the highest order limit encodes information on the geometry of local structures as a two-component isoperimetric problem, while the second level describes the spatial distribution of components in global minimizers. We provide a sharp rigorous derivation of the asymptotic limit, both for minimizers and in the context of Gamma-convergence. Geometrical descriptions of limit configurations are derived. The main difficulties are hidden in the optimal solution of two-component isoperimetric problem: compared to binary systems, not only it lacks an explicit formula, but, more crucially, it can be neither concave nor convex on parts of its domain.

ERIC WOOLGAR, University of Alberta
Uniqueness problems for quasi-Einstein equations

I will discuss some uniqueness problems for the Einstein equations and its close relatives, especially the quasi-Einstein equation (but also the Ricci flow equation). These problems are motivated by the study of black hole solutions of general relativity, but are all posed for metrics of Riemannian signature. The problems are partly or entirely open.

KAZUO YAMAZAKI, Texas Tech University
Recent developments for convex integration on fluid PDEs

I will review recent developments on the technique of convex integration applied to PDEs in hydrodynamics (mainly stochastic ones) that has led to various non-uniqueness results in both deterministic and stochastic cases. Examples of equations include the Navier-Stokes equations, magnetohydrodynamics system, surface quasi-geostrophic equations, and transport (continuity) equation, etc.

DEPING YE, Memorial University of Newfoundland
Mou He Fang Gai: A legend over thousands years

The study of Mou He Fang Gai (also known as the bicylinder) dated back to Archimedes, Hui Liu, Chongzhi Zu, Geng Zu etc. In particular, the old Chinese mathematicians Chongzhi Zu and Geng Zu took use of Mou He Fang Gai to find the explicit formula of the volume of 3-dimensional balls. This is one of the milestone results in the (Chinese) mathematical history.

In this talk, I will briefly explain how to use Mou He Fang Gai to find the formula for 3-dimensional ball. Motivated by the construction of Mou He Fang Gai, I will also talk about how to develop a new polarity (and hence a new family of convex bodies) for sets in the n-dimensional space. This new polarity naturally defines many useful notions parallel to those for convex bodies. In particular, I will explain how to get a new Blaschke-Santalo type inequality.

QI S. ZHANG, UC Riverside
Log gradient estimates of the heat equation on manifolds.

We will first review a number of log gradient estimates for the heat equation by Li-Yau, Hamilton and Perelman and their application in geometry and topology. Next we present a recent result showing a sharp Li Yau estimate for all compact manifolds and impossibility of sharp estimate for all noncompact manifolds, answering an open question by several people. Recent extension by X. Song, L. Wang, M. Zhu and Z will also be mentioned.