
Applying mathematics to operations research and real life problems
Application des mathématiques à la recherche opérationnelle et aux problèmes de la vie réelle
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We examine the relevance of quantum strategies for deterrence related game theory.

It was shown that in fundamental games like the Magic Square, quantum algorithms provide superior results to those of classical algorithms. To model more complex scenarios, we explore the extension of the Magic Square game to higher dimensions. We observe that quantum algorithms remain superior to classical game theory at higher dimensions in the Magic Square game.

Building on this observation on a basic and fundamental game, we investigate the extension of this result on more realistic games such as the Prisoner's Dilemma and how quantum strategies can help players coordinate their decisions to ensure an optimal decision for both in which they will not betray each other (i.e. they will be deterred)

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Fitting Linear Ordinary Differential Equation and Machine Learning Models using Matrix Frechet derivatives with application in

Using a trace formula recently introduced in [Nguyen, 2022] for the Frechet derivative of an analytic matrix function, we revisit the problem of fitting a model whose state variables are governed by a system of linear differential equations. Applications include fitting the equation for radioactivity in blood samples [Jennrich and Bright, 1976] and fitting financial time series. The trace formula allows us to use a derivative-based solver for both problems. We also discuss other applications of Frechet derivative in numerical calculations, including finding zeros of a function involving matrix exponential in machine learning [Sustik and Dhillon, 2012] and finding the Riemannian center of mass (in computer vision) [Chakraborty and Vemuri, 2019].

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The Maths Behind Trustworthy Intelligent Vision Systems

Nowadays, there is an increased use of AI-based technologies in applications ranging from intelligent agents to autonomous vehicles, from Industry 4.0 to Society 5.0. One of the main challenges posed by all these new-generation intelligent systems is their trustworthiness. Hence, this work presents the mathematical modelling that underlies trustworthy intelligent systems and studies its operational use in context of advanced computer vision applications.

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A computational stochastic approach for determination of coordinated air defence firing strategies

Coordinated air defence involves the determination of a firing strategy across a group of air defence units employing surface-to-air missiles to counter an air raid. A key objective for a firing strategy is to achieve a desired hit probability against each

threat in the raid whilst minimising total missile expenditure to counter the raid. Instrumental to this is the adoption of shoot-look-shoot firing policies where possible. The problem is subject to time and resource constraints and is further complicated by uncertainties in threat behaviours as the raid progresses. This paper presents an overview of a computational stochastic method developed for solving this problem in context of naval task group air defence. This method involves the construction of decision trees composed of firing options arising from projections of the threats through the engagement zones for the air defence units. Each firing option captures a possible engagement for a defending unit firing a salvo of one or more missiles against a threat at a specified time. Associated with each firing option is an engagement assessment performed using precomputed missile engagement zones that provide intercept times and hit probabilities for given launch conditions. The decision tree is searched to yield a Pareto efficient boundary for hit probability versus missile expenditure, from which an optimal firing strategy can be obtained. The methodology is now being applied in a fast running ship stationing model to complement physics-based Monte Carlo models for investigating task group air defence.