# Matrices and Operators (Bilingual Session) <br> Matrices et opérateurs (session bilingue) <br> (Org: Ludovick Bouthat (Université Laval), Javad Mashreghi (Université Laval) and/et Frédéric Morneau-Guérin <br> (TÉLUQ/Laval)) 

## LUDOVICK BOUTHAT, Université Laval <br> Weighted averages of $\ell^{p}$ sequences: New generalizations of Hardy's inequality

The classical Hardy inequality states that $\sum_{n=1}^{\infty}\left(\frac{a_{1}+\cdots+a_{n}}{n}\right)^{p} \leq\left(\frac{p}{p-1}\right)^{p} \sum_{n=1}^{\infty} a_{n}^{p}$, where $\left(a_{n}\right)_{n \geq 1}$ is a given sequence of nonnegative real number. The objective of this talk is to present three new Hardy-type inequalities in which the arithmetic mean over a sequence of nonnegative real numbers is replaced by some weighted arithmetic mean over some nested subsets of the given sequence of numbers. One of these inequalities stems from a calculation in a recent paper on semi-infinite matrices of Bouthat and Mashreghi.

## JAVAD MASHREGHI, Laval University <br> Carleson measures in classical function spaces

We study some "one-box condition" for Carleson measures in Hardy, Bergman and Dirichlet spaces. In particular, we show that a finite measure $\mu$ on the unit disk is a Carleson measure for the Dirichlet space if it satisfies the Carleson one-box condition $\mu(S(I))=O(\phi(|I|))$, where $\phi:(0,2 \pi) \rightarrow(0, \infty)$ is an increasing function such that $\int_{0}^{2 \pi} \phi(x) / x d x<\infty$. We also show that the integral condition on $\phi$ is sharp.
Joint work with O. El-Fallah, K. Kellay, T. Ransford.

## FRÉDÉRIC MORNEAU-GUÉRIN, Universite TELUQ

Poids de convolution sur $\ell^{2}$
Il est bien connu que l'espace $L^{p}$ pondéré sur un groupe localement compact est stable par rapport à la convolution si la fonction de pondération satisfait une certaine inégalité de convolution. Il existe plusieurs contre-exemples montrant que cette condition suffisante n'est pas nécessaire. Cependant, pour une classe de groupes, à savoir les groupes abéliens discrets, aucun contre-exemple n'est connu. Il subsiste donc une possibilité que l'inégalité de convolution caractérise vraiment la stabilité de la convolution pour les espaces $L^{p}$ pondérés sur ces groupes. Dans cet exposé, nous étudions d'une part cette inégalité et, dans le cas $p=2$, nous la réinterprétons à la lumière de la théorie des opérateurs et dans le contexte de la théorie des espaces de Hilbert à noyau reproductible. D'autre part, nous esquisserons quelques tentatives infructueuses de générer des contre-exemples.

MATHIAS NEUFANG, Carleton University and University of Lille
Non-commutative Fejer theorems, and Arens regularity of the projective tensor product of $C^{*}$-algebras
We present solutions to several problems concerning crossed products and tensor products of operator algebras. The common theme is our use of completely bounded module maps.
We prove that a locally compact group $G$ has the approximation property (AP) if and only if a non-commutative Fejer theorem holds for the associated $C^{*}$ - or von Neumann crossed products. We deduce that the AP always implies exactness. This generalizes a result of Haagerup-Kraus, and answers a question by Li. We also answer a problem of Bedos-Conti on discrete $C^{*}$-dynamical systems, and one by Anoussis-Katavolos-Todorov on bimodules over the group von Neumann algebra $V N(G)$ for all locally compact groups $G$ with the AP. We also obtain a version of our von Neumann algebraic Fejer theorem for discrete quantum groups. (Joint work with J. Crann and S. Kazemi.)

It has been open for about 40 years to characterize when the projective Banach space tensor product $A \otimes_{\gamma} B$ of two $C^{*}$-algebras $A$ and $B$ is Arens regular. We solve this problem for arbitrary $C^{*}$-algebras: Arens regularity is equivalent to $A$ or $B$ having the Phillips property; hence, it is encoded in the geometry of the algebras. For von Neumann algebras $A$ and $B$, we conclude that $A \otimes_{\gamma} B$ is Arens regular only if $A$ or $B$ is finite-dimensional. This does not generalize to non-selfadjoint operator algebras. For a commutative $C^{*}$-algebra $A$, we prove that the centre of $\left(A \otimes_{\gamma} A\right)^{* *}$ is Banach algebra isomorphic to the extended Haagerup tensor product $A^{* *} \otimes_{e h} A^{* *}$.

## MARCU-ANTONE ORSONI, University of Toronto

Dominating sets, spectral estimates and null-controllability.
Let $(\Omega, \mu)$ be a measure space and let $\mathcal{F} \subset L^{p}(\Omega, \mu)$ be a subspace of holomorphic functions. A measurable set $E$ is said to be dominating for $\mathcal{F}$ if there exists a constant $C_{E}>0$ such that

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\int_{\Omega}|f|^{p} d \mu \leq C_{E} \int_{E}|f|^{p} d \mu, \forall f \in \mathcal{F}
$$

In this talk, I will start giving estimates of the sampling constant $C_{E}$ for Bergman spaces and Fock type spaces. Then, I will explain how this question is related to certain spectral inequalities that play a central role in the null controllability of parabolic equations. Based on joint works with A. Hartmann and S. Konaté.

PIERRE-OLIVIER PARISÉ, University of Hawaii at Manoa
Divergence of Taylor Series in de Branges-Rovnyak Spaces
In this talk, I will present sufficient conditions for the existence of a function in a given de Branges-Rovnyak space for which the Taylor series is unbounded in norm or diverges to infinity in norm. The result is a consequence of a refined version of the boundedness principle established by Müller and Vrsovsky. This is a joint work with Thomas Ransford.

## RAJESH PEREIRA, University of Guelph

Linear maps which preserve convex sets and their geometric and spectral properties
Let $C$ be a convex subset of a vector space $V$ and let $\left\{x_{i}\right\}$ be a finite collection of points in $C$. We consider the set of all linear maps from $V \rightarrow V$ that preserve both $C$ and all of the points $\left\{x_{i}\right\}$. Specific choices of $C$ and $\left\{x_{i}\right\}$ give the set of correlation matrices, the set of doubly stochastic matrices and the set of positive linear maps. We explore some geometric properties of these convex sets and some spectral properties of matrices in these convex sets.

## WILLIAM VERREAULT, Université Laval <br> Nonlinear expansions in reproducing kernel Hilbert spaces

I will introduce an expansion scheme in reproducing kernel Hilbert spaces, which as a special case covers the celebrated Blaschke unwinding series expansion for analytic functions, also known as adaptive Fourier decomposition. The expansion scheme can also be generalized to cover certain reproducing kernel Banach spaces. I will discuss convergence results for this series expansion, which has been a major question with the Blaschke unwinding, as well as a few concrete applications and examples.
This is based on joint work with Javad Mashreghi.

