**Hopf Algebras and Related Topics**
**Algèbres de Hopf et sujets connexes**
(Org: **Yevgenia Kashina** (DePaul University, Chicago), **Mikhail Kotchetov** (Memorial University), **Mitja Mastnak** (Saint-Mary's University) and/et **Yorck Sommerhäuser** (Memorial University))

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**MARCELO AGUIAR**, Cornell University
*The Eckmann-Hilton argument in duoidal categories*

We will go over the basics of duoidal categories, illustrating with a number of examples. As monoidal categories provide a context for monoids, duoidal categories provide one for duoids and bimonoids. Our main goal is to discuss a number of versions of the classical Eckmann-Hilton argument which may be formulated in this setting. As an application we will obtain the commutativity of the cup product on the cohomology of a bimonoid with coefficients in a duoid, an extension of a familiar result for group and bialgebra cohomology.

The talk borrows on earlier work in collaboration with Swapneel Mahajan on the foundations of duoidal categories (2010). The main results are from ongoing work with Javier Coppola. We also rely on work of Richard Garner and Ignacio López-Franco (2016).

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**RYAN AZIZ**, Université Libre de Bruxelles
*Generalize Yetter-Drinfeld Modules and Center of Biactegories*

We study the notion of the $E$-center of bi-actegory $\mathcal{Z}_E(\mathcal{M})$ where $\mathcal{M}$ is a $(\mathcal{C},\mathcal{D})$-biactegory (or bimodule category) relative to an op-monoidal functor $E:\mathcal{C}\to\mathcal{D}$. We apply the theory to $\mathcal{M}={}_A\mathrm{Mod}$, $\mathcal{C}={}_H\mathrm{Mod}$, and $\mathcal{D}={}_K\mathrm{Mod}$, and $E\cong C\otimes_H - : {}_H\mathrm{Mod}\to{}_K\mathrm{Mod}$, where $A$ is a $(H,K)$-bicomodule algebra and $C$ is a $(K,H)$-bimodule coalgebra. Under the condition that $A$ is an $H$-Galois object, we show that the $E$-center of ${}_A\mathrm{Mod}$ is equivalent to the category of generalized Yetter-Drinfeld modules as introduced by Canaepeel, Militaru, and Zhu, generalizing the similar well-known result for the usual Yetter-Drinfeld modules.

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**STEFAN CATOIU**, DePaul University
*Recent developments in the theory of generalized derivatives via algebra*

We outline a few recent developments in the theory of generalized derivatives: 1) the solution to the subject's main problem on the equivalence between the Peano and Riemann derivatives, going back to Khintchine in 1927; 2) the solution of the problem of classifying the equivalences between any two generalized Riemann derivatives, going back to Ash in 1967; 3) the solution to the GGR conjecture on the equivalence between the Peano and sets of generalized Riemann derivatives, formulated by Ginchev, Guerragio and Rocca in 1998; and 4) the solution to a question by G. Benkart in 2021, on the Leibniz Rule for generalized Riemann derivatives. All these recent proofs involved some sort of algebra: linear algebra, polynomial algebra, graded algebra, group algebra, and coalgebra. The talk is based on joint work with J. Marshall Ash, William Chin, Marianna Csörnyei and Hajrudin Fejzić.

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**KENNY DE COMMER**, Vrije Universiteit Brussel
*Doi-Koppinen modules and quantized Harish-Chandra modules*

A (left) Doi-Koppinen datum consists of a bialgebra $H$ together with a right $H$-comodule algebra $A$ and a left $H$-module coalgebra $C$. A Doi-Koppinen module is then a left $A$-module which is at the same time a right $C$-comodule, such that the module and comodule structure are compatible in a natural way. Natural Doi-Koppinen data can be constructed from right coideal subalgebras in bialgebras. In this talk, we will revisit the theory of Doi-Koppinen modules for particular coideal subalgebras obtained from Letzter's quantum symmetric pairs, and will show that the associated Doi-Koppinen modules provide a natural framework for the quantization of Harish-Chandra modules associated to real semisimple Lie groups. If we have time, we will explain how in this setting, the Doi-Koppinen modules acquire a natural monoidal structure, based on a theorem due to Takeuchi. This is joint work with J.R. Dzokou Talla.

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**HONGDI HUANG**, Rice University
*Twisting of graded quantum groups and comodule algebras*

One particualr interesting deformation of a Hopf algebra is its 2-cocycle twist. On another hand, a graded algebra can be deformated by its grade automorphisms, which is called Zhang twist. In this talk, we will introduce the sufficient conditions how to deform a Hopf algebra by Zhang twist. In addition, we will systematically describe a Zhang twist of a Hopf algebra as a 2-cocycle twist; and a Zhang twist of a comodule algebra as a 2-cocycle twist over the Manin's universal quantum groups.

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**ELLEN KIRKMAN**, Wake Forest University
*McKay matrices for finite-dimensional Hopf algebras*

Let H be a finite dimensional Hopf algebra over an algebraically closed field of characteristic zero with simple modules $\mathsf{S}_1, \ldots, \mathsf{S}_m$, and let V be a fixed H-module. The McKay matrix $\mathsf{M}_\mathsf{V}$ of V encodes the multiplicities of each $\mathsf{S}_j$ as a composition factor of each $\mathsf{S}_i \otimes \mathsf{V}$. Steinberg showed that for $\mathsf{H} = \mathbb{C}G$ the eigenvalues and the eigenvectors of $\mathsf{M}_\mathsf{V}$ are related to characters, and further results in characteristic $p$ were obtained by Grinberg, Huang and Reiner. We prove general results about McKay matrices, their eigenvalues, and their (left and right) (generalized) eigenvectors by using the coproduct and the characters of simple and projective H-modules. We illustrate these results for the Drinfeld double $\mathsf{D}_n$ of the Taft algebra for $n$ odd and $n \geq 3$. This is joint work with Georgia Benkart, Rekha Biswal, Van Nguyen, and Jieru Zhu.

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**JEAN-SIMON PACAUD LEMAY**, Macquarie Universirty
*Lifting Trace with Hopf Algebras and Hopf Monads*

A Hopf algebra $H$ in a symmetric monoidal category $\mathbb{X}$ has the special ability of lifting many desirable structures and properties of $\mathbb{X}$ to MOD$(H)$, the category of $H$-modules. Indeed, MOD$(H)$ will be a symmetric monoidal category, and if $\mathbb{X}$ is closed, or star-autonomous, or even compact closed, then MOD$(H)$ will be as well. The antipode of $H$ plays a crucial role in lifting these structures. In this talk, I will explain how Hopf algebras also have the ability of lifting traces. Traced monoidal categories, introduced by Joyal, Street and Verity, are symmetric monoidal categories equipped with a trace operator, which generalizes the classical notion of the trace of matrices in linear algebra. Traced monoidal categories have many applications in mathematics, quantum foundations, and computer science. If $\mathbb{X}$ is a traced monoidal category, then for a Hopf algebra $H$, MOD$(H)$ will be a traced symmetric monoidal category. In particular, this means that the trace of an $H$-module morphism is again an $H$-module morphism. We will also consider the special cases of compact closed categories (where the trace is given by duals), or when the monoidal product is a product (where the trace is given by fixpoints) or a coproduct (where the trace is given by iteration). We will also discuss how this fact also generalizes to the notion of Hopf monads, in the sense of Bruguières, Lack, and Virelizier.

This is joint work with Masahito Hasegawa, and is based on our paper: arXiv:2208.06529

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**KAYLA ORLINSKY**, University of Southern California
*Second indicators of the fusion category $\mathcal{C}(G, H)$ where $G$ is a Coxeter group and $H$ is a reflection subgroup of $G$*

This is an ongoing joint project with Peter Schauenburg. In 2009, Guralnick and Montgomery showed that if $G$ is a finite real reflection group, then $D(G)$–the Drinfel'd double of $G$ over an algebraically closed field $k$ of characteristic not $2$–is totally orthogonal. That is, all irreps of $D(G)$ have indicator $+1$. Using the notation of [Schauenburg 2016], we explore several cases where the second indicator of the simple objects of the group-theoretical fusion category $\mathcal{C}(G, H)$ are all nonnegative where $G$ is a finite Coxeter group and $H$ is a reflection subgroup of $G$.

**BAHRAM RANGIPOUR**, University of New Brunswick
*Toward the primary conjecture*

Hopf cyclic cohomology was invented by A. Connes and H. Moscovici to compute the local index cocycle associated to a hypoelliptic operator on the frame bundle twisted by the group of diffeomorphisms. The goal was to compute the cocycle in the Gelfand-Fuks cohomology of formal vector fields. To the speaker's knowledge, the only computation so far is done by the inventors in degree 1 to show the index cocycle is 1. There is a conjecture that states that the cocycle is made of primary classes. Toward this direction we associate a sequence of coalgebras to the Lie algebra of formal vector fields on the Euclidean space . We also introduce a Hopf algebra that acts on all coalgebras in the sequence. We compute the Hopf cyclic cohomology of some of the coalgebras to make sure the path is the right one. This is a collaboration with Serkan Sutlu.

**SEAN SANFORD**, The Ohio State University
*Non-Split Tambara-Yamagami Categories over the Reals*

In 1998, Tambara and Yamagami classified all split fusion categories with a certain simple set of fusion rules that occur naturally as categories of complex representations of finite groups. When taking real representations, irreducible representations can be real a.k.a. *split*, or they can be complex or quaternionic, a.k.a. *non-split*. For example, $\mathrm{Rep}_{\mathbb{R}}(Q_8)$ contains a quaternionic irreducible of dimension 4. In a recent paper with J. Plavnik and D. Sconce, we have extended the classification to now include such non-split irreducibles. I will give many examples, and along the way I will discuss some of the complications involved in working with fusion categories over the reall numbers.

**JOOST VERCRUYSSE**, Université Libre de Bruxelles
*A Hopf category of Frobenius algebras*

A well-known result of Sweedler tells that the category of algebras can be enriched over coalgebras, by considering the universal measuring coalgebra between two algebras as the Hom-object between them. Another way of stating this result, is that the category of algebras can be given a semi-Hopf category structure. By a similar construction, one can build a universal measuring coalgebra $C(A, B)$ between any two Frobenius algebras $A$ and $B$ (being not just compatible with the algebra structure but also with their coalgebra (or Frobenius) structure). A remarkable observation is that in this way we do not just obtain a semi-Hopf category structure but even a Hopf category, meaning that there exists an anti-coalgebra morphism from $C(A, B)$ to $C(B, A)$ satisfying a natural antipode property. In particular, the universal acting bialgebra on a Frobenius algebra is always Hopf, which generalizes the known result that any (endo)morphism of Frobenius algebras is invertible.

This is based on joint works with Ana Agore and Alexey Gordienko, and with Paul Grosskopf.

A. Agore, A. Gordienko and J. Vercruysse, $V$-universal Hopf algebras (co)acting on $\Omega$-algebras, Commun. Contemp. Math. 25 (2023), Paper No. 2150095, 40 pp.

E. Batista, S. Caenepeel and J. Vercruysse, Hopf categories, Algebr. Represent. Theory 19 (2016), 1173-1216.

P. Grosskopf and J. Vercruysse, Free and co-free constructions for Hopf categories, arXiv:2305.03120.

P. Grosskopf and J. Vercruysse, The Hopf category of Frobenius algebras, in preparation.

**XINGTING WANG**, xingting.wang@howard.edu
*Twisting Manin's universal quantum groups and comodule algebras*

In this talk, we will discuss the homological properties invariant under Morita-Takeuchi equivalence. In particular, we consider the infinite coaction of the Manin's universal quantum groups on an AS-regular algebra. As a consequence, the AS-regularity is invariant under 2-cocycle twist. This is joint work with Hongdi Huang, Van C. Nguyen, Charlotte Ure, Kent B. Vashaw, and Padmini Veerapen.

**YILONG WANG**, Yanqi Lake Beijing Institute of Mathematical Sciences and Applications
*Modular tensor categories from SL(2,Z) representations*

Modular data is an essential invariant of a modular tensor category, and they enjoy various algebraic properties such as rationality, congruence property and Galois symmetry. In this talk, we use the algebraic properties of modular data, or to be more precise, of the modular group representations to study modular tensor categories. As an example, we will talk about our result on the classification of transitive modular tensor categories and the symmetrization of congruence representations of SL(2,Z). This talk is based on joint works with Siu-Hung Ng, Samuel Wilson and Qing Zhang.

**RUI XIONG**, University of Ottawa
*Structure algebras, Hopf algebroids and oriented cohomology of a group*

In this talk, we present our work on proving that the structure algebra of a Bruhat moment graph of a finite real root system is a Hopf algebroid with respect to the Hecke and the Weyl actions. We introduce new techniques and apply them to linear algebraic groups, generalized Schubert calculus, and the combinatorics of Coxeter groups and finite real root systems. Our results have interesting implications for the natural Hopf-algebra structure on the algebraic oriented cohomology of Levine-Morel and for computing the Hopf-algebra structure of "virtual cohomology" of dihedral groups $I_2(p)$, where $p$ is an odd prime.

**QING ZHANG**, Purdue University
*Super-modular categories from near-group centers*

A super-modular category is a unitary pre-modular category with Müger center equivalent to the symmetric unitary category of super-vector spaces. The modular data for a super-modular category gives a projective representation of the group: $\Gamma_\theta < \mathrm{SL}(2,\mathbb{Z})$. Adapting work of Ng-Rowell-Wang-Wen, Cho-Kim-Seo-You computed modular data from congruence representations of $\Gamma_\theta$ using the congruence subgroup theorem for super-modular categories of Bonderson-Rowell-Wang-Z and the minimal modular extension theorem of Reutter-Johnson-Freyd. They found two classes of previously unknown modular data for rank 10 super-modular categories. We show that these data are realized by modifying the Drinfeld centers of near-group fusion categories associated with the groups $\mathbb{Z}/6$ and $\mathbb{Z}/2 \times \mathbb{Z}/4$. This is based on joint work with Eric Rowell and Hannah Solomon.