Early Career Research in Number Theory
Recherche en début de carrière en théorie des nombres
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JÉRÉMY CHAMPAGNE, University of Waterloo
Diophantine approximation by linear forms with angular restrictions

Diophantine approximation by linear forms can be understood as follows: Given real numbers $\alpha_1, ..., \alpha_n$, one seeks integers $q_1, ..., q_n, p$ such that the value of the linear form $q_1\alpha_1 + \cdots + q_n\alpha_n - p$ is *small* in comparison to the size of $q_1, ..., q_n$ (consider the uncanny smallness of $2\pi + e - 9$, for example).

In this talk, we consider the case where the point $\mathbf{x} = (q_1, ..., q_n, p)$ is required to make a bounded angle with a prescribed subspace of $\mathbb{R}^{n+1}$. We give an optimal lower bound for the exponent of approximation in this context, which surprisingly only depends on the dimension of the prescribed subspace.

This is joint work with Damien Roy.

SOURABHASHIS DAS, University of Waterloo
On the number of irreducible factors with a given multiplicity in function fields

Let $k \geq 1$ be a natural number and $f \in \mathbb{F}_q[t]$ be a monic polynomial. Let $\omega_k(f)$ denote the number of distinct monic irreducible factors of f with multiplicity k. In this talk, we show that the function $\omega_1(f)$ has a normal order $\log(\text{deg}(f))$ and also satisfies the Erdös-Kac Theorem. We also show that the functions $\omega_k(f)$ with $k \geq 2$ do not have normal order. Such results are obtained by studying the first and the second moments of $\omega_k(f)$ which we explain in brief. This is joint work with Ertan Elma, Wentang Kuo, and Yu-Ru Liu.

MIHIR DEO, University of Ottawa
Factorization of unbounded p-adic L-functions

Let F_α, F_β be two power series over a finite extension of the field of p-adic numbers $\mathbb{Q}_p$ satisfying certain interpolation formulae. Suppose further that the coefficients of the power series have unbounded denominators satisfying certain growth condition. In this talk, we will discuss the decomposition of F_α and F_β into linear combinations of two power series with integral coefficients. We use p-adic Hodge theory, in particular the theory of Wach modules and Perrin-Riou's p-adic regulator to construct a logarithmic matrix (in the spirit of Sprung and Lei-Loeffler-Zerbes) which is used in the factorization. This is an extension of a result of Büyükboduk-Lei and is a part of my ongoing project which deals with the factorization of two variable p-adic L-function attached to a small slope Bianchi modular form constructed by Williams.

JASON FANG AND ANTON MOSUNOV, University of Waterloo
A Lower Bound for the Area of the Fundamental Region of a Binary Form

Let

$$F(x, y) = \prod_{k=1}^{n} (\delta_k x - \gamma_k y)$$

be a binary form of degree $n \geq 1$, with complex coefficients, written as a product of n linear forms in $\mathbb{C}[x, y]$. Let

$$h_F = \prod_{k=1}^{n} \sqrt{|\gamma_k|^2 + |\delta_k|^2}$$

denote the height of F and let A_F denote the area of the fundamental region $\mathcal{D}_F$ of F:

$$\mathcal{D}_F = \left\{(x,y) \in \mathbb{R}^2 \colon |F(x,y)| \leq 1\right\}.$$

We prove that $h_F^{2/n} A_F \geq \left(2^{1+(r/n)}\right)\pi$, where r is the number of roots of F on the real projective line $\mathbb{RP}^1$, counting multiplicity.

NIC FELLINI, Queen's University
Variations of a conjecture of Ankeny-Artin-Chowla

A famous conjecture of Ankeny, Artin and Chowla relates the class number of a real quadratic field $\mathbb{Q}(\sqrt{p})$ with p a prime congruent to $1 \bmod 4$ with its fundamental unit $\varepsilon = (t + u\sqrt{p})/2$ via a congruence $\bmod\, p$. In particular, the Ankeny-Artin-Chowla (AAC) conjecture states that u is not divisible by p. The significance of their conjecture lies in the fact that it provides an arithmetic way of computing the class number of $\mathbb{Q}(\sqrt{p})$ for p a prime congruent to $1 \bmod 4$. We will discuss the history and techniques of their work as well as show that there are further connections with Fermat quotients and Wieferich style congruences. This is joint work with M. Ram Murty.

SAMPRIT GHOSH, University of Toronto
Higher Euler-Kronecker coefficients

The coefficients that appear in the Laurent series of Dedekind zeta functions and their logarithmic derivatives, about $s = 1$, are mysterious and seem to contain a lot of arithmetic information. Although the residue and the constant term have been widely studied, not much is known about the higher coefficients. In this talk, we present some results about these coefficients $\gamma_{K,n}$ that appear in the Laurent series expansion of $\frac{\zeta_K'(s)}{\zeta_K(s)}$ about $s = 1$, where K is a global field. For example, when K is a number field, we prove, under GRH, (if d_K is the absolute discriminant of K)

$$\gamma_{K,n} \ll (\log(\log(|d_K|))^{n+1}$$

TONY HADDAD, Université de Montréal
A coupling for the prime factors of a random integer

The sizes of large prime factors for a random integer N sampled uniformly in $[1,x]$ are known to converge in distribution to a Poisson-Dirichlet process $\mathbf{V} = (V_1, V_2, \ldots)$ as $x \to \infty$. In 2002, Arratia constructed a coupling of N and $\mathbf{V}$ satisfying $\mathbb{E}\sum_i |\log P_i - (\log x)V_i| = O(\log\log x)$ where $P_1 P_2 \cdots$ is the unique factorization of N with $P_1 \geqslant P_2 \geqslant \cdots$ being all primes or ones. He conjectured that there exists a coupling for which this expectation is $O(1)$.

I will present a modification of his coupling which proves his conjecture, and show that $O(1)$ is optimal. As a corollary, I will provide a simpler proof of the arcsine law in the average distribution of divisors proved by Deshouillers, Dress and Tenenbaum in 1979. This is joint work with Dimitris Koukoulopoulos.

TING HAN HUANG, Concordia University
Special values of triple product p-adic L-functions and p-adic Abel-Jacobi maps

In 2013, H. Darmon and V. Rotger proved the so-called p-adic Gross-Zagier formula, which relates the value of the triple product p-adic L-function for Hida families at a certain balanced classical weight, to an image of the p-adic Abel-Jacobi map of the generalized diagonal cycle.

In this talk, I will present a generalization of their result to finite slope families.

We first introduce the construction of the triple product p-adic L-function by F. Andreatta and A. Iovita. Then we explain the Abel-Jacobi map, the explicit computation of which involves A. Besser's finite polynomial cohomology theory.

In the end, we will show how to relate the two objects, and hence prove the p-adic Gross-Zagier formula.

DANIEL JOHNSTONE, University of Toronto

A construction of some Stable Transfer Operators

In this talk I will discuss some recent progress on the construction of Stable Transfer Operators $\mathfrak{S}^{\phi}$ associated to an L-embedding $\phi : {}^{L}\mathsf{GL}_n \to {}^{L}G$. While many such explicit constructions remain out of reach, I will discuss a manner in which this general case can be largely reduced to an understanding of the case of an embedding $\phi : {}^{L}S \to {}^{L}G$ for a maximal torus S of GL_n. This method relies on building sections for the transfer maps associated to the embeddings ${}^{L}S \to {}^{L}\mathsf{GL}_n$ for each maximal torus S of GL_n, in addition to a related family of related maps. As a guiding example, I will give a construction of the transfer associated to the diagonal embedding ${}^{L}\mathsf{GL}_n \to {}^{L}\mathsf{GL}_n \times \mathsf{GL}_n$ which ought to be considered as a type of non-abelian convolution on the space of orbital integrals on the Steinberg-Hitchin base of GL_n.

MARTI ROSET JULIA, McGill University

The Gross–Kohnen–Zagier theorem via p-adic uniformization

Let S be a set of rational primes of odd cardinality containing infinity and a rational prime p. We can associate to S a Shimura curve X defined over $\mathbb{Q}$. The Gross–Kohnen–Zagier theorem states that certain generating series of Heegner points of X are modular forms of weight $3/2$ valued in the Jacobian of X. We will state this theorem and outline a new approach to prove it using the theory of p-adic uniformization. This is joint work in progress with Lea Beneish, Henri Darmon and Lennart Gehrmann.

VALERIYA KOVALEVA, CRM/Universite de Montreal

Correlations of the Riemann Zeta on the critical line

In this talk we will discuss the behaviour of the Riemann zeta on the critical line, and in particular, its correlations in various ranges. We will prove a new result for correlations of squares, where shifts may be up to size $T^{6/5-\varepsilon}$. We will also explain how this result relates to Motohashi's formula for the fourth moment, as well as the moments of moments of the Riemann Zeta and its maximum in short intervals.

MALORS ESPINOSA LARA, University of Toronto

A Symbol from Beyond Endoscopy

To carry out the process of Beyond Endoscopy, as proposed by Langlands around 2000, for Gl(2) for general number fields it is necessary to be able to write the Hecke Sign Character in a very explicit form. For the rational numbers this is achieved via the Kronecker Symbol, but for other number fields it is not so straightforward how to do it in the way needed for Beyond Endoscopy.

In a joint work with Melissa Emory, Debanjana Kundu and Tian An Wong, we developed a way to do this by modifying in a precise way several local Hilbert Symbols and thus recovering the sign character in the explicit way we require. In this talk I will talk about this construction, some examples of it and why it was needed as well as some questions we are not yet able to settle related to it.

MATT OLECHNOWICZ, University of Toronto

Distribution of preperiodic points in one-parameter families

Let f_t be a one-parameter family of rational maps (of degree at least 2) defined over a number field K. We show that for all t outside of a set of natural density zero, every K-rational preperiodic point of f_t is the specialization of some $K(T)$-rational preperiodic point of f. Assuming a weak form of the Uniform Boundedness Conjecture, we also find the average number of

K-rational preperiodic points of any family, and give some examples where this holds unconditionally. This talk will not assume any prior knowledge of arithmetic dynamics.

LIAM OROVEC, University of Waterloo
Small Univoque Bases

For a positive number q, we say (ε_i) is a q-expansion for x provided, $x = \sum_{i=1}^{\infty} \frac{\varepsilon_i}{q^i}$. Working over the alphabet $\mathcal{A} = \{0, 1, \ldots, M\}$ we look at finding, given a fixed positive real number x, the smallest base $q_s(x)$ for which x has a unique $q_s(x)$-expansion.

We will first establish the result for $x = 1$. Then using relations between the representation of 1 under base $q_s(x)$ and the possible unique representation of real numbers we determine whether $q_s(x) \leq q_s(1)$ which will aid us in calculating the desired value.

This is a generalization of the work of D. Kong who established the results for $M = 1$. The study of such bases is important as most x have an infinite number of representations under an arbitrary base q.

SHUYANG SHEN, University of Toronto
On Irreducible Trinomials

Consider the family of monic trinomials $f(x) = x^n + lx^m + a$ in $\mathbb{Z}[x]$. We present a classification of reducible polynomials in this family, and discuss the Galois groups of irreducible trinomials. This is joint work with Kumar Murty.

MATTHEW SUNOHARA, University of Toronto
On stable transfer operators and functorial transfer kernels

Langlands introduced stable transfer operators as a fundamental part of his proposal of Beyond Endoscopy. They are intended to be used in comparisons of his proposed refinements of stable trace formulas, in an analogous role to that of endoscopic transfer operators in the theory of Endoscopy. The existence of stable transfer operators for real groups is readily established, but there remains the problem of obtaining explicit formulas for their distributional kernels, the so-called stable transfer factors or functorial transfer kernels. We will discuss stable transfer between tori, complex groups, and work in progress on stable transfer from real groups to tori, which would include a generalisation of the Gelfand-Graev formula for stable discrete series characters of SL(2).

YASH TOTANI, University of Waterloo
On the problem of representing integers by quadratic forms

Historically, one of the most extensively studied problem in the theory of quadratic forms is finding the number of representations of an integer by a quadratic form. A well known result in this area is Jacobi's four square theorem which gives explicit formulas for the number of representations of an integer n as a sum of four squares. Another interesting insight was given by Fred van der Blij in a 1952 paper, where he gives exact formulas for the number of representations for all three equivalence classes of quadratic forms Q of discriminant -23. In this talk, we look at a generalization of the above result for other values of the discriminant $D < 0$, such that $\mathbb{Q}(\sqrt{D})$ has class number three, using the theory of modular forms.

WILLIAM VERREAULT, Université Laval
Sums of arithmetic functions running on factorials

5

We examine the behavior of common arithmetic functions at factorial arguments. For various arithmetic functions f, the asymptotic behavior of $f(n!)$, $\sum_{n\leq N} f(n!)$, and $f(n!)/f((n-1)!)$ is obtained. An analogue of Chowla's conjecture for factorial arguments is also investigated.

This is joint work with Jean-Marie De Koninck.