
Computational Aspects in Low-Dimensional Topology and Contact Geometry

Aspects computationnels de la topologie de basse dimension et de la géométrie de contact

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DROR BAR-NATAN, University of Toronto, Mathematics

Computing the Zombian of an Unfinished Columbarium

The zombies need to compute a quantity, the zombian, that pertains to some structure - say, a columbarium. But unfortunately (for them), a part of that structure will only be known in the future. What can they compute today with the parts they already have to hasten tomorrow's computation?

That's a common quest, and I will illustrate it with a few examples from knot theory and with two examples about matrices - determinants and signatures. I will also mention two of my dreams (perhaps delusions): that one day I will be able to reproduce, and extend, the Rolfsen table of knots using code of the highest level of beauty.

See also <http://drorbn.net/ott23>.

AGNESE BARBENSI, University of Melbourne

Hypergraphs for multiscale cycles in structured data

In this talk we will explore techniques to include homology generators in the persistent homology pipeline, and we will see how these can be used to infer geometric and structural information on the underlying system, with a focus on spatial curves and biological applications.

DANIELE CELORIA, University of Melbourne

GridPyM: a Python module to handle grid diagrams

Grid diagrams are a combinatorial version of classical link diagrams, widely used in theoretical, computational and applied knot theory. Motivated by questions from (bio)-physical knot theory, we introduce GridPyM, a Sage compatible Python module that handles grid diagrams. GridPyM focuses on generating and simplifying grids, and on modelling local transformations between them.

JIE CHEN, McMaster University

The concordance group of flat knots

Virtual knots were introduced by Kauffman, and they represent knots in thickened surfaces up to stable equivalence. Each virtual knot determines a flat knot, which is the homotopy class of the immersed curve in the surface. Turaev raised a list of questions regarding sliceness of flat knots and concordance classes of long flat knots in 2004. I will talk about my work on the concordance group of flat knots based on calculated results in FlatKnotInfo and conjectures suggested by the tabulation.

JAMES HALVERSON, Northeastern

Searching for ribbons with machine learning

We apply Bayesian optimization and reinforcement learning to a problem in topology: the question of when a knot bounds a ribbon disk. This question is relevant in an approach to disproving the four-dimensional smooth Poincaré conjecture; using our programs, we rule out many potential counterexamples to the conjecture. We also show that the programs are successful in detecting many ribbon knots in the range of up to 70 crossings.

ANDRÁS JUHÁSZ, Oxford

The unknotting number, hard unknot diagrams, and Reinforcement Learning

We have developed a Reinforcement Learning agent based on the IMPALA architecture that often finds minimal unknotting trajectories for a knot diagram up to 200 crossings. We have used this to determine the unknotting number of 57k knots. We then took diagrams of connected sums of such knots with oppositely signed signatures, where the summands were overlaid. The agent has found unknotting trajectories involving several crossing changes that result in hyperbolic knots. Based on this, we have shown that, given knots K and K' that are not 2-bridge, there is a diagram of their connected sum and $u(K) + u(K')$ unknotting crossings such that changing any one of them results in a prime knot. As a by-product, we have obtained a dataset of 2.6 million distinct hard unknot diagrams; most of them under 35 crossings. Assuming the additivity of the unknotting number, we can determine the unknotting number of 43 at most 12-crossing knots for which the unknotting number is unknown. This is joint work with Taylor Applebaum, Sam Blackwell, Alex Davies, Thomas Edlich, and Marc Lackenby.

HOMAYUN KARIMI, McMaster University

Mock Seifert matrices and unoriented algebraic concordance

In this talk, we describe the concordance properties of signature and determinant invariants for knots in thickened surfaces. If $K \subset \Sigma \times I$ is $\mathbb{Z}/2$ null-homologous and slice, we show that its signatures vanish and its determinants are perfect squares. A mock Seifert matrix is an integral square matrix representing the Gordon-Litherland form of a pair (K, F) , where K is a knot in a thickened surface and F is an unoriented spanning surface for K . Using these matrices, we introduce a new notion of unoriented algebraic concordance, as well as a new group denoted $m\mathcal{G}^{\mathbb{Z}}$ and called the unoriented algebraic concordance group. This group is abelian and infinitely generated. Mock Seifert matrices can also be used to define new invariants, such as the mock Alexander polynomial and mock Levine-Tristram signatures. These invariants are applied to questions about virtual knot concordance, crosscap numbers, and Seifert genus for knots in thickened surfaces. This talk is based on joint works with Hans U. Boden.

PATRICIA SORYA, Université du Québec à Montréal (UQÀM)

Characterizing slopes: explicit bounds for satellite knots

A slope p/q is said to be characterizing for a knot if the homeomorphism type of its p/q -Dehn surgery determines the knot up to isotopy. The existence of a lower bound for $|q|$ that guarantees p/q is characterizing for a given knot has been established in recent work. I am currently collaborating with Laura Wakelin to determine this bound explicitly for an infinite family of satellite knots.

MISHA TYOMKIN, Dartmouth College

On numbers associated with a strong Morse function

Morse function on a manifold M is called strong if all its critical points have different critical values. Given a strong Morse function f and a field \mathbb{F} we construct a bunch of elements of \mathbb{F} , which we call Bruhat numbers (they're defined up to sign). More concretely, Bruhat number is written on each bar in the barcode of f (a.k.a. Barannikov decomposition). It turns out that if homology of M over \mathbb{F} is that of a sphere, then the product of all the numbers is independent of f . We then construct the barcode and Bruhat numbers with twisted (a.k.a. local) coefficients and prove that the mentioned product equals the Reidemeister torsion of M . In particular, it's again independent of f . This way we link Morse theory to the Reidemeister torsion via barcodes. Time permitting, we will also discuss how parametric Morse theory comes into play. Based on a joint work with Petya Pushkar.

ZACHARY WINKELER, Smith College

Spectral sequence computations in knot Floer homology

A defining feature of the knot Floer homology of a knot $K \subset S^3$ is the spectral sequence converging to the Heegaard Floer homology of S^3 . By studying the behavior of the Legendrian invariants λ^\pm under this spectral sequence, we can obstruct the existence of decomposable Lagrangian cobordisms between Legendrian knots. I will talk about recent work in this direction, including computational aspects and potential applications of similar ideas to other spectral sequences.

This talk is based on joint work with Mitchell Jubeir, Ina Petkova, Noah Schwartz, and C.-M. Michael Wong.

THOMAS WOLF, Brock University

TurboKnots

The package TurboKnots started as a tool box for working with knot diagrams to generate random knots, simplify diagrams by performing 'beneficial' Reidemeister moves, extracting prime knots that may overlap, performing flype moves and pass moves that reduce or preserve the number of crossings and by compactifying diagrams. Knots with up to 15 crossings can be identified instantly. The talk will mention coloring and unknotting computations of all knots with up to 15 crossings and comment on the efficiency of its HOMFLYPT polynomial computations.