Arithmetic aspects of automorphic forms
Aspects arithmétiques des formes automorphes

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ANTONIO CAUCHI, Concordia University

Towards new Euler systems for automorphic Galois representations

The construction of Euler systems for Galois representations associated to automorphic forms often relies on the existence of Rankin-Selberg integrals which calculate the corresponding L-function. I will discuss a new Rankin-Selberg integral, which represents a twist of the degree 5 L-function of cusp forms on GSp_4, and its application to the study of the arithmetic of the standard Galois representation associated to cusp forms on GSp_4. This is joint work with Armando Gutierrez.

HENRI DARMON, McGill

Generalised Hecke eigenvectors

If M is a module over a Hecke algebra and $v \in M$ is a simulatenous eigenvector for the Hecke operators, a *generalised eigenvector* attached to v is a an element $v' \in M$ satisfying

$$T_\ell v' = a_\ell v' + a'_\ell v.$$

The scalars a'_ℓ often carry rich arithmetic information. When M is the space of forms of weight one, they are logarithms of algebraic numbers and are a key to explicit class field theory for real quadratic fields. I will discuss the case where M is a space of modular forms of weight two with fourier coefficients in $\mathbb{Z}/p\mathbb{Z}$, where these quantites appear to be related to classes in K-theory considered by Beilinson and Flach.

This is an account of work in progress with Alice Pozzi.

CÉDRIC DION, Université Laval

Refined conjectures on Fitting ideals of Selmer groups

Fix an odd prime number p and let K_n be the nth layer of the cyclotomic $\mathbb{Z}_p$-extension of $\mathbb{Q}$. Kim and Kurihara showed that the Pontryagin dual of the Selmer group over K_n attached to an elliptic curve E with good ordinary reduction at p is generated by the Mazur-Tate element at level K_n. When the reduction is supersingular at p, they show that the same result holds, up to an explicit error term. In this talk, we discuss generalization of these results for the Selmer group of E over the $\mathbb{Z}_p^2$-extension of an imaginary quadratic field where p splits.

RYLAN GAJEK-LEONARD, Union College

Iwasawa invariants of nonordinary modular forms

This talk will outline a method for computing the (analytic) Iwasawa invariants of cuspidal newforms with $a_p = 0$ in terms of the associated sequence of Mazur-Tate elements. The connection between Mazur-Tate elements and p-adic L-functions is well-known for weight two modular forms, but the relation is less clear at higher weights. In the $a_p = 0$ case, we use Pollack's decomposition of the p-adic L-function to construct explicit lifts of Mazur-Tate elements to the full Iwasawa algebra. By studying the behavior of these lifts upon projection to layer n, we relate the Iwasawa invariants of Mazur-Tate elements to those of the corresponding p-adic L-functions. Corollaries include a relation between the Iwasawa invariants attached to certain p-congruent pairs of modular forms and a description of the p-adic valuation of critical L-values for modular forms with $a_p = 0$.

EYAL GOREN, McGill University
Foliations on Shimura varieties in positive characteristic

In joint work with E. De Shalit (Hebrew U) we developed a theory of foliations on Shimura varieties in positive characteristic, offering as "case studies" the examples of Hilbert modular varieties and unitary Shimura varieties. Those reveal intimate connections with modular forms mod p, Shimura varieties with parahoric level structure, inseparable morphisms and deformation theory. I will provide an overview, using two particular examples: Hilbert-Blumenthal surfaces and Picard modular surfaces.

JEFFREY HATLEY, Union College
Vanishing anticyclotomic μ-invariants for non-ordinary modular forms

Let $E_{/\mathbf{Q}}$ be an elliptic curve and p a prime such that $E[p]$ is irreducible as a $G_{\mathbf{Q}}$-module. A fundamental conjecture (due to Greenberg and Perrin-Riou) states that the Iwasawa μ-invariant(s) associated to E over the cyclotomic $\mathbf{Z}_p$-extension of $\mathbf{Q}$ must vanish. Despite many focused efforts, this conjecture is still wide open. One may extend this conjecture to more general modular forms and more general $\mathbf{Z}_p$-extensions of number fields. In this talk, we discuss work (joint with Antonio Lei) which establishes some cases of this conjecture over anticyclotomic $\mathbf{Z}_p$-extensions of imaginary quadratic fields.

DEBANJANA KUNDU, Fields Institute
λ-invariant stability in Families of Modular Galois Representations

Consider a family of modular forms, all of whose residual $\pmod p$ Galois representations are isomorphic. It is well-known that their corresponding Iwasawa λ-invariants may vary. We will discuss this variation from a quantitative perspective, providing lower bounds on the frequency with which these λ-invariants grow or remain stable. This is joint work with Jeff Hatley.

JEF LAGA, Princeton University
Rational torsion on abelian surfaces with quaternionic multiplication

Mazur classified all possible rational torsion subgroups of elliptic curves over $\mathbb{Q}$. In joint work with Ciaran Schembri, Ari Shnidman and John Voight, we put strong constraints on the torsion subgroup of a class of abelian surfaces whose geometric endomorphism algebra is large, namely an indefinite quaternion algebra. The proof uses quaternion arithmetic, Neron models, and the modularity of abelian surfaces of GL_2-type.

HEEJONG LEE, University of Toronto
Emerton-Gee stacks for GSp_4 and Serre weight conjectures

In the Langlands program, we want to construct a certain correspondence between automorphic representations and Galois representations. The meaning of this correspondence can be explained in terms of the L-functions. However, one can also ask how the structure of one side is reflected on the other side. Serre weight conjectures explicitly explain that how (Serre) weights of the automorphic side and the ramification behavior on the Galois side are related. During the first half of my talk, I will discuss the Serre weight conjectures and their relation to local Galois representations. This will motivate us to understand certain Galois deformation rings. Then I will discuss Emerton-Gee stacks (which allows a more geometric approach to Galois representations) and local models of Le-Le Hung-Levin-Morra (which can describe parts of Emerton-Gee stacks explicitly), as well as their generalizations to the group GSp_4.

ADAM LOGAN, Government of Canada
A conjectural uniform construction of many rigid Calabi-Yau threefolds

Given a rational Hecke eigenform f of weight 2, Eichler-Shimura theory gives a construction of an elliptic curve over $\mathbb{Q}$ whose associated modular form is f. Mazur, van Straten, and others have asked whether there is an analogous construction for Hecke eigenforms f of weight $k>2$ that produces a variety for which the Galois representation on its etale H^{k-1} (modulo classes of cycles if k is odd) is that of f. In weight 3 this is understood by work of Elkies and Schütt, but in higher weight it remains mysterious, despite many examples in weight 4. In this talk I will present a new construction based on families of K3 surfaces of Picard number 19 that recovers many existing examples in weight 4 and produces almost 20 new ones.

MOHAMMADREZA MOHAJER, University of Ottawa
Linear relations of p-adic periods of 1-motives

1-periods are complex numbers arising from degree 1 Betti-de Rham comparison isomorphism or from Deligne 1-motives. Due to Wüstholz, Huber and other research works, Kontsevich-Zagier period conjecture is known for these periods. In our research, we are aiming to draw p-adic analogies with the well-established results that are known for these periods. Specifically, in this talk, we will explore the p-adic periods of curve type. Our main goal is to study the transcendence and linear relations of these p-adic numbers. We will begin by introducing the formalism of p-adic periods where it provides us a tool to state the p-adic period conjecture and the p-adic version of subgroup theorem. We will then move on to the p-adic periods of 1-motives with good reduction which arise from crystalline-de Rham realisations and we will compare them with those p-adic periods coming from p-adic integration theory.

KATHARINA MUELLER, Universite Laval
On the Iwasawa invariants of BDP-Selmer groups and BDP p-adic L-functions

Let f_1 and f_2 be weight two Hecke cusp forms with isomorphic residual Galois representations. Let K be an imaginary quadratic field and assume that the generalized Heegner hypothesis holds for the pairs (f_1,K) and (f_2,K). Let K_∞/K be the anticyclotomic $\mathbb{Z}_p$-extension. We will analyze the relation between the Iwasawa invariants of the BDP-Selmer groups over K_∞ and the BDP p-adic L-fucntions for f_1 and f_2.

This is joint work with Antonio Lei and Jiacheng Xia.

SIDDARTH SANKARAN, University of Manitoba
Arithmetic Siegel-Weil formiulas for zero dimensional varieties.

The arithmetic Siegel-Weil formula is a conjectural identity, due to Kudla, that predicts relations between certain arithmetic 'special' cycles on Shimura varieties and derivatives of Eisenstein series. In the zero-dimensional case, notable examples include work of Kudla-Rapoport-Yang, Howard, and Andreatta-Goren-Howard-Madapusi; these results are ultimately proved by an explicit computation.

This talk is part of an ongoing effort to understand these results from a more conceptual point of view; after reviewing the formula in a simple case, we will place these results in a more general context, and explain how the arithmetic Siegel-Weil formula can be seen as an application of the usual Siegel-Weil formula. The main novelty is the introduction of a p-adic "Green function" in the zero-dimensional setting, mirroring a construction of Kudla at the archimedean place.

JIACHENG XIA, Université Laval
The convergence problem in Kudla's modularity conjectures

Kudla's modularity conjectures generalize the classical Gross–Kohnen–Zagier theorem for modular curves and Heegner points to Shimura varieties for orthogonal and unitary groups and special cycles of arbitrary codimensions. These conjectures were confirmed by Yuan–Zhang–Zhang and Liu assuming convergence of the generating series of special cycles over totally real fields and CM fields. To fill this gap of convergence, in a joint work in progress with Qiao He, we prove an algebraicity result of these generating series which is a crucial step in our approach towards the convergence problem.