
Analytic Number Theory and L-functions
Théorie analytique des nombres et fonctions L
(Org: **Chantal David** (Concordia) and/et **Yu-Ru Liu** (Waterloo))

AMIR AKBARY-MAJDABADNO, University of Lethbridge

Discrepancy estimates for the value-distribution of the quadratic twists of automorphic L-functions

We describe an upper bound for the discrepancy of the distribution of the values (at a point on the edge of the critical strip) of the twists of a fixed automorphic L -function with quadratic Dirichlet characters. Our result can be considered as an automorphic analogue of a result of Lamzouri, Lester, and Radziwill for the logarithm of the Riemann zeta function. Our estimate is conditional on certain expected bounds on the local parameters of L -functions which is known to be true for $GL(1)$ and $GL(2)$. This is a joint work with Alia Hamieh (UNBC).

VIVIAN KUPERBERG, Stanford University

Sums of singular series and the distribution of primes

The Hardy–Littlewood conjectures predict that the number of times a specific configuration of length k appears in the primes is governed by constants known as singular series. Via applications of the Hardy–Littlewood conjectures, sums of these constants have many applications in the study of the distribution of primes. We will discuss some results about refined sums of singular series and their applications, including sums over large sets and the tail of the distribution of primes in short intervals.

WANLIN LI, CRM

On the vanishing of twisted L-functions of elliptic curves over function fields

We investigate the vanishing at $s = 1$ of the twisted L -functions of elliptic curves E defined over the rational function field $\mathbb{F}_q(t)$, for twists by Dirichlet characters of prime order $\ell \geq 3$, from both a theoretical and numerical point of view. In the case of number fields, it is predicted that such vanishing is a very rare event, and our numerical data seems to indicate that this is also the case over function fields for non-constant curves. For constant curves, we prove that if there is one χ_0 such that $L(E, \chi_0, 1) = 0$, then there are infinitely many. Finally, we provide some examples which show that twisted L -functions of constant elliptic curves over $\mathbb{F}_q(t)$ behave differently than the general ones. This is joint work with Antoine Comeau-Lapointe, Chantal David, and Matilde Lalin.

ALLYSA LUMLEY, Université de Montréal

Selberg's Central limit theorem for quadratic dirichlet L-functions over function fields

In this talk we will discuss the logarithm of the central value $L\left(\frac{1}{2}, \chi_D\right)$ in the symplectic family of Dirichlet L -functions associated with the hyperelliptic curve of genus g over a fixed finite field \mathbb{F}_q in the limit as $g \rightarrow \infty$. Unconditionally, we show that the distribution of $\log |L\left(\frac{1}{2}, \chi_D\right)|$ is asymptotically bounded above by the full Gaussian distribution of mean $\frac{1}{2} \log \deg(D)$ and variance $\log \deg(D)$, and also $\log |L\left(\frac{1}{2}, \chi_D\right)|$ is at least 94.27% Gaussian distributed. Assuming a mild condition on the distribution of the low-lying zeros in this family, we obtain the full Gaussian distribution.

ANTON MOSUNOV, University of Waterloo

On the representation of integers by binary forms defined by means of the relation $(x + yi)^n = R_n(x, y) + J_n(x, y)i$

Let F be a binary form with integer coefficients, degree $d \geq 3$ and non-zero discriminant. Let $R_F(Z)$ denote the number of integers of absolute value at most Z which are represented by F . In 2019 Stewart and Xiao proved that $R_F(Z) \sim C_F Z^{2/d}$

for some positive number C_F . We compute C_{R_n} and C_{J_n} for the binary forms $R_n(x, y)$ and $J_n(x, y)$ defined by means of the relation

$$(x + yi)^n = R_n(x, y) + J_n(x, y)i,$$

where the variables x and y are real.

MIKE RUBINSTEIN, University of Waterloo

Differential equations related to averages of the k -th divisor function

Keating, Rodgers, Roditty-Gershon, and Rudnick have given a conjecture for the asymptotic behaviour of the mean square of sums of the k -th divisor numbers over short intervals, and have proven formulas for the analogous problem over $\mathbb{F}_q[t]$. I will discuss their work and describe determinantal and differential equations related to their formulas.

NAHID WALJI, University of British Columbia

On the decomposition of automorphic symmetric power L -functions for $GL(3)$ and $GL(4)$

We investigate the conjectural decomposition of symmetric power lifts of automorphic representations for $GL(3)$, via the study of the corresponding L -functions. We consider the asymptotic behaviour of exterior, symmetric power, and Rankin–Selberg L -functions, and show that under various assumptions about automorphy and cuspidality, we can bound the maximum number of isobaric summands for the k th symmetric power lift. In particular, we show it is bounded above by 3 for $k \geq 7$, and bounded above by 2 when $k \geq 19$ with k congruent to 1 (mod 3). We will also discuss the analogous problem for $GL(4)$.