Convex geometry and partial differential equations
Géométrie convexe et équations aux dérivées partielles
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MIN CHEN, University of Science and Technology of China
Alexandrov-Fenchel type inequalities in the sphere

The Alexandrov-Fenchel inequalities for quermassintegrals in the Euclidean spaces are classical topics in differential geometry and convex geometry. The corresponding problem in non-convex hypersurfaces in space forms has gained much interest recently but remains largely unsettled. The application of curvature flows to prove the geometric inequalities is nowadays classical. In this talk, I will introduce a recent work about the Alexandrov-Fenchel inequalities in the sphere by employing suitable curvature flows.

PENGFEI GUAN, McGill University
A weighted gradient estimate for nonlinear PDE associated to the Christoffel-Minkowski problem

The classical Christoffel-Minkowski problem is an inverse type problem in convex geometry. One wants to find a convex body with prescribed kth area measure. At one end $k = 1$, it's the Christoffel problem, at the other end $k = n$ it's the Minkowski problem. There is also a L^p version of this type of problem: the intermediate L^p Christoffel-Minkowski problem $0 < p$. The problem corresponds to a degenerate Hessian type equation on S^n. We will discuss a recent work on the weighted gradient estimate and its application to the existence and convexity of solution to this type of equations.

JULIAN HADDAD, Universidade Federal de Minas Gerais
Explicit representations of isotropic measures in extremal positions

It is known since the work of F. John in 1948, that if the unit euclidean ball is the ellipsoid of minimal volume containing a convex body K, then there is a decomposition of the identity given by a centered isotropic measure supported in the set of contact points.

In this work, we present a constructive proof of this measure, and propose an algorithm to compute the weights of the decomposition when the contact points are finite.

Based in a joint work with F. Baêta.

STEVEN HOEHNER, Longwood University
Asymptotic expected T-functionals of random polytopes with applications to L_p surface areas

Asymptotic formulas are proved for the expected T-functional of the random convex hull of independent and identically distributed random points sampled from the Euclidean unit sphere in $\mathbb{R}^n$ according to a positive continuous density. As an application, the approximation of the sphere by random polytopes in terms of L_p surface area differences is discussed.

HAN HUANG, Georgia Institute of Technology
Average Case Analysis of Gaussian Elimination with Partial Pivoting

The Gaussian Elimination with Partial Pivoting (GEPP) is a classical algorithm for solving systems of linear equations. Although in specific cases the loss of precision in GEPP due to roundoff errors can be very significant, empirical evidence strongly suggests that for a typical square coefficient matrix, GEPP is numerically stable.

We obtain a (partial) theoretical justification of this phenomenon by showing that, given the random $n \times n$ standard Gaussian coefficient matrix A, the growth factor of the Gaussian Elimination with Partial Pivoting is at most polynomially large in n with probability close to one. This implies that with probability close to one the number of bits of precision sufficient to solve $Ax = b$ to m bits of accuracy using GEPP is $m + O(\log n)$, which improves an earlier estimate $m + O(\log^2 n)$ of Sankar, and which we conjecture to be optimal by the order of magnitude. We further provide tail estimates of the growth factor which can be used to support the empirical observation that GEPP is more stable than the Gaussian Elimination with no pivoting.

This talk is based on a joint work with Konstantin Tikhomirov.

JIUZHOU HUANG, McGill University
Flow by powers of Gauss curvature in space forms

Flow by powers of Gauss curvature is an important extrinsic flow in differential geometry. The flow in Euclidean space was considered by many authors and many interesting results had been got there. In this talk, I will talk about the problem in space forms and establish the analogous results to that of the Euclidean case. This is based on a recent work joint with Min Chen.

DYLAN LANGHARST, Kent State University
Measure Theoretic Minkowski's Existence Theorem and Projection Bodies

The Brunn-Minkowski Theory has seen several generalizations over the past century. Many of the core ideas have been generalized to measures. With the goal of framing these generalizations as a measure theoretic Brunn-Minkowski theory, we prove the Minkowski existence theorem for a large class of Borel measures with density, denoted by Λ': for ν a finite, even Borel measure on the unit sphere and $\mu \in \Lambda'$, there exists a symmetric convex body K such that

$$d\nu(u) = c_{\mu,K} dS_{\mu,K}(u),$$

where $c_{\mu,K}$ is a quantity that depends on μ and K and $dS_{\mu,K}(u)$ is the surface area-measure of K with respect to μ. Examples of measures in Λ' are homogeneous measures (with $c_{\mu,K} = 1$) and probability measures with continuous densities (e.g. the Gaussian measure). We will also consider measure dependent projection bodies $\Pi_\mu K$ by classifying them and studying the isomorphic Shephard problem: if μ and ν are even, homogeneous measures with density and K and L are symmetric convex bodies such that $\Pi_\mu K \subset \Pi_\nu L$, then can one find an optimal quantity $\mathcal{A} > 0$ such that $\mu(K) \leq \mathcal{A}\nu(L)$? Among other things, we show that, in the case where $\mu = \nu$ and L is a projection body, $\mathcal{A} = 1$.

AIJUN LI, Zhejiang University of Science and Technology
On the sine polarity and the L_p-sine Blaschke-Santaló inequality

This talk is dedicated to study the sine version of polar bodies and establish the L_p-sine Blaschke-Santaló inequality for the L_p-sine centroid body.

The L_p-sine centroid body $\Lambda_p K$ for a star body $K \subset \mathbb{R}^n$ is a convex body based on the L_p-sine transform, and its associated Blaschke-Santaló inequality provides an upper bound for the volume of $\Lambda_p^\circ K$, the polar body of $\Lambda_p K$, in terms of the volume of K. Thus, this inequality can be viewed as the "sine cousin" of the L_p Blaschke-Santaló inequality established by Lutwak and Zhang. As $p \to \infty$, the limit of $\Lambda_p^\circ K$ becomes the sine polar body $K^\diamond$ and hence the L_p-sine Blaschke-Santaló inequality reduces to the sine Blaschke-Santaló inequality for the sine polar body. The sine polarity naturally leads to a new class of convex bodies $\mathcal{C}_e^n$, which consists of all origin-symmetric convex bodies generated by the intersection of origin-symmetric closed solid cylinders. Many notions in $\mathcal{C}_e^n$ are developed, including the cylindrical support function, the supporting cylinder, the cylindrical Gauss image, and the cylindrical hull. Based on these newly introduced notions, the equality conditions of the sine Blaschke-Santaló inequality are settled.

YOUJIANG LIN, Chongqing Technology and Business University
The Petty projection inequalities

In this talk, we introduce some recent results about Petty projection inequalities. Particularly, we define the Petty projection body about star bodies and prove the Orlicz-Petty projection inequality about star bodies. Moreover, we introduce some functional inequalities with respect to affine isoperimetric inequalities.

CHUNYAN LIU, School of Mathematics and Statistics,Huazhong University of Science and Technology
Ulam floating functions

We extend the notion of Ulam floating sets from convex bodies to Ulam floating functions. We use the Ulam floating functions to derive a new variational formula for the affine surface area of log-concave functions.

STEPHANIE MUI, NYU Courant
On the L^p dual Minkowski problem for $-1 < p < 0$

The L^p dual curvature measure was introduced by Lutwak, Yang, and Zhang in 2018. The associated Minkowski problem, known as the L^p dual Minkowski problem, asks about existence of a convex body with prescribed L^p dual curvature measure. This question unifies the previously disjoint L^p Minkowski problem with the dual Minkowski problem, two famous open questions in convex geometry. We prove the existence of a solution to the L^p dual Minkowski problem for the case of $q < 1 + p$, $-1 < p < 0$, and $p \neq q$ for even measures.

PETER PIVOVAROV, University of Missouri
Isoperimetric inequalities for polar Lp centroid bodies

I will discuss an extension of the Lutwak-Zhang theorem for the polar of Lp centroid bodies to the case p in (0,1). Joint work with R. Adamczak, G. Paouris, and P. Simanjuntak.

MICHAEL ROYSDON, Tel Aviv University
Extensions of Zhang' inequality

This talk will detail two recent papers concerning the Rogers- Shephard difference body inequality and Zhang's inequality for various classes of measures. The covariogram of a convex body with respect to a measure plays an essential role in the proofs of each of these inequalities. In particular, we will discuss a variational formula concerning the covariogram resulting in a measure theoretic version of the projection body operator. If time permits, we will discuss how these results imply some reverse isoperimetric inequalities.

KATERYNA TATARKO, University of Waterloo
L_p Steiner formula and its coefficients

In this talk, we explore L_p Steiner formula for the L_p affine surface area. We introduce the coefficients that arise in this formula that we call L_p-Steiner quermassintegrals and discuss their properties. It turns out that they possess some nice properties. In particular, they are new valuations on the set of convex bodies. Based on joint works with E. Werner.

SUDAN XING, University of Alberta
On Multiple Lp-curvilinear-Brunn-Minkowski inequality

In this talk, the extension of the curvilinear summation for bounded Borel measurable sets to the L_p space for multiple power parameter $\bar{\alpha} = (\alpha_1, \cdots, \alpha_{n+1})$ when $p > 0$ will be introduced. Based on this $L_{p,\bar{\alpha}}$-curvilinear summation for sets and concept of $compression$ of sets, the $L_{p,\bar{\alpha}}$-curvilinear-Brunn-Minkowski inequality for bounded Borel measurable sets and its normalized version are established. Furthermore, by utilizing the hypo-graphs for functions, we enact a brand new proof of

$L_{p,\bar{\alpha}}$ Borell-Brascamp-Lieb inequality, as well as its normalized version, for functions containing the special case of L_p Borell-Brascamp-Lieb inequality through the $L_{p,\bar{\alpha}}$-curvilinear-Brunn-Minkowski inequality for sets. We also propose the multiple power $L_{p,\bar{\alpha}}$-supremal-convolution for two functions together with its properties. Moreover, we introduce the definition of the surface area originating from the variation formula of measure in terms of the $L_{p,\bar{\alpha}}$-curvilinear summation for sets as well as $L_{p,\bar{\alpha}}$-supremal-convolution for functions together with their corresponding Minkowski type inequalities and isoperimetric inequalities for $p \geq 1$, etc. This talk is based on the joint work with Dr. Michael Roysdon.

PING ZHONG, University of Wyoming

The Brown measure of the sum of a free random variable and Voiculescu's circular element or its elliptic deformation

The circular element is the most important example of non-normal random variable used in free probability, and its Brown measure is the uniform measure in the unit disk. The circular element has connection to asymptotics of non-normal random matrices with i.i.d. entries. We obtain a formula for the Brown measure of the addition $x_0 + c$ of an arbitrary free random variable x_0 and circular element c, which is known to be the limit empirical spectral distribution of deformed i.i.d. random matrices.

Generalizing the case of circular and semi-circular elements, we also consider g, a family of elliptic deformations of c, that is $*$-free from x_0. Possible degeneracy then prevents a direct calculation of the Brown measure of $x_0 + g$. We instead show that the whole family of Brown measures of operators $x_0 + g$ are the push-forward measures of the Brown measure of $x_0 + c$ under a family of self-maps of the complex plane, which could possibly be singular. We calculate density formulas for various interesting examples.

This work generalizes previous results of Bordenave-Caputo-Chafaï, Hall-Ho, and a joint work with Ho. The main results offer potential applications to various deformed random matrix models. Our method is based on a Hermitian reduction and subordination functions in free probability.